Linear models with time-varying parameters in gretl: comparing different approaches

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Background

- Interest in **changes in DGP** over time:
  - Structural breaks (Chow, 1960; Bai and Perron, 2003)
  - Smooth Transitions (Chan and Tong, 1986)
  - Markov-switching models (Hamilton, 1989)
  - **Time-varying-parameters** (Cooley and Prescott, 1976; Harvey, 1990)
- Huge body of literature (Hamilton, 2020)
- Hot topic: WoS finds 25 articles
  - with *"time-varying parameters"* in title/abstract
  - Year: 2022-2023
  - Journals: *Econometrica*, *JoE*, *JAE*
- Blasques et al. (2023): themed issue *Time-varying parameters in econometrics* of *JoE*

Baseline model

We will consider the model

$$y_t = \boldsymbol{x}_t' \boldsymbol{\beta}_t + e_t \quad \text{with} \quad t = 1, \ldots, T$$

- y_t is a scalar outcome
- $\boldsymbol{x}_t$ is K-dimensional vector of covariates
- $\boldsymbol{\beta}_t$ is K-dimensional vector of **time-varying** parameters
- e_t is an idiosyncratic component

Estimators of $\boldsymbol{\beta}_t$

- Flexible Least Squares (FLS)
- Varying-Coefficients (VC)
- ML via Kalman Filter (KF)
- Kernel-based estimator (KB)

Flexible Least Squares (Kalaba and Tesfatsion, 1989)

Given

$$\boldsymbol{u}_t = \boldsymbol{\beta}_t - \boldsymbol{\beta}_{t-1},$$

find the sequence $\hat{\boldsymbol{\beta}}_t$ that minimizes the criterion function (similar in spirit to the HP filter):

$$C(\mu, T) = \sum_{t=1}^{T} \left(y_t - \boldsymbol{x}'_t \boldsymbol{\beta}_t\right)^2 + \mu \sum_{t=2}^{T} \boldsymbol{u}'_t \boldsymbol{u}_t$$

where μ is a smoothness parameter such that $\mu \to \infty \Longrightarrow \boldsymbol{\beta}_t \to \hat{\boldsymbol{\beta}}_{OLS}$.

- $\hat{\boldsymbol{\beta}}_t$ can be obtained analytically or by a recursion
- Not in gretl (yet)
- No confidence bands

The VC estimator (Schlicht, 2021)

Consider the model:

$$e_t = y_t - \boldsymbol{x}_t' \boldsymbol{\beta}_t$$
$$\boldsymbol{u}_t = \boldsymbol{\beta}_t - \boldsymbol{\beta}_{t-1}$$

and assume

$$E(e_t) = 0 \quad \text{and} \quad E(e_t^2) = \sigma^2$$
$$E(\boldsymbol{u}_t) = \boldsymbol{0} \quad \text{and} \quad E(\boldsymbol{u}_t \boldsymbol{u}_t') = diag(\phi_1^2, \dots, \phi_K^2)$$

The VC estimator minimizes

$$C(\gamma_1, \dots, \gamma_K, T) = \sum_{t=1}^{T} e_t^2 + \sum_{t=2}^{T} \boldsymbol{u}_t' W \boldsymbol{u}_t \quad \text{with} \quad W = diag(\gamma_1, \dots, \gamma_K)$$

with $\gamma_k = \hat{\sigma}^2 / \hat{\phi}_k^2$

The VC estimator (cont'd)

- VC is obtained by iterations until convergence
- VC are moment estimators
- Some parameter can be time-fixed (restricted $\phi_k^2 \to 0$)
- When $e_t \sim N(0, \sigma^2)$, VC coincides with ML estimator
- VC equals the KF estimator when states are independent RWs

Kernel-based Least Squares (Giraitis et al., 2021)

The time-varying OLS estimator

$$\hat{\beta}_t^{KB} = \left(\sum_{j=1}^{T} b_{H,|j-t|} \boldsymbol{x}_j \boldsymbol{x}_j' \right)^{-1} \left(\sum_{j=1}^{T} b_{H,|j-t|} \boldsymbol{x}_j y_j \right) \tag{1}$$

with

- $b_{H,|j-t|} = K(\frac{|j-t|}{T^h})$
- $K(\cdot)$ is a kernel function (Gaussian, Epanechnikov, ...)
- Rolling-window is a special case (rectangular kernel)
- h is the bandwidth parameter
- Data-driven bandwidth selection (Lucchetti and Valentini, in press)
- Extension to IV

Kalman Filter

The state-space representation is

$$\begin{aligned} \text{Measurement Equation:} \quad & y_t = \boldsymbol{x}'_t \boldsymbol{\beta}_t + e_t \\ \text{Transition Equation:} \quad & \boldsymbol{\Phi}(L)\boldsymbol{\beta}_t = \boldsymbol{\mu} + \boldsymbol{u}_t \end{aligned}$$

ML estimation under suitable assumptions

- For the transition equation, we consider the standard case $\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{u}_t$
- $e_t \sim N(0, \sigma_e^2)$
- $E(\boldsymbol{u}_t) = \mathbf{0}$ and $E(\boldsymbol{u}_t \boldsymbol{u}'_t) = \Sigma_{u,t}$
- $E(e_t \boldsymbol{u}'_t) = \mathbf{0}'$
- Initialization via KB

Open source routines

- Gretl

Estimator	Native	Package	Author(s)	Last update
KF	✓	✗		
KB	✗	`ketvals`	Lucchetti R. & Valentini F.	Jan 2023
VC	✗	`TVC`	Schlicht E.	Aug 2022

- Other open-source software:

Estimator	Software	Package	Author(s)	Last update
FLS	`Fortran`	`FLS`	Tesfatsion L.	NA
	`R`	`fls`	Lewis B.	Apr 2010
	`R`	`fls`	Bagdonas G.	Nov 2019
KF	`R`	several	...	...

- KF in `R` via several options, see Tusell (2011) for a quite old review

Simulated data - DGP

We generate data from

$$y_t = x_{1,t}\beta_{1,t} + x_{2,t}\beta_{2,t} + x_{3,t}\beta_{3,t} + \varepsilon_t$$

with

- $t = 1, \ldots, 256$
- $x_{j,t} \sim N(0, 1)$
- $\beta_{j,t} = \sum_{s=1}^{t} N(0\ ,\ 0.01)$
- $\varepsilon_t \sim N(0\ ,\ 0.49)$

Figure: Simulated data

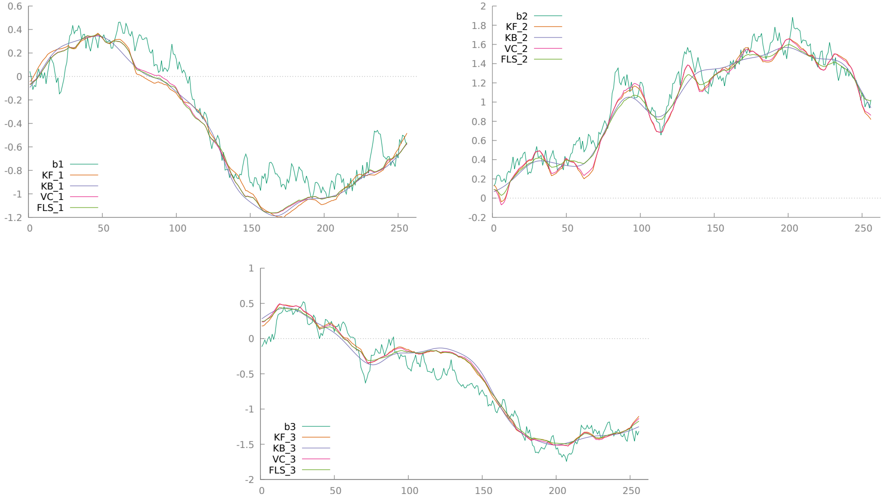


Table: CPU time (sec)

		$T = 128$	$T = 256$	$T = 512$
$k = 2$	KF	0.312	0.843	1.893
	KBA	0.183	0.908	5.455
	KB	0.018	0.018	0.031
	VC	1.537	8.462	94.63
	FLS	0.257	0.015	0.010
$k = 4$	KF	1.561	4.392	36.21
	KBA	0.218	1.386	13.19
	KB	0.008	0.007	0.033
	VC	42.66	114.7	2561
	FLS	0.009	0.019	0.013
$k = 8$	KF	23.50	57.62	144.2
	KBA	0.581	3.047	26.73
	KB	0.005	0.018	0.060
	VC	545.2	5033	45561
	FLS	0.011	0.014	0.022

Application: Okun's law

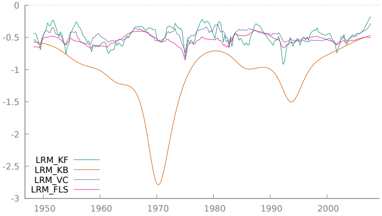
- Trade-off between unemployment and output
- Quarterly data from the St.Louis FED database
- u_t, unemployment rate
- $\tilde{y}_t$, output gap (HP-filter on GDP)
- ECM model

$$\Delta u_t = \beta_{1,t} + \beta_{2,t} u_{t-1} + \beta_{3,t} \tilde{y}_{t-1} + \beta_{4,t} \Delta \tilde{y}_t + \beta_{5,t} \Delta \tilde{y}_{t-1} + \eta_t$$

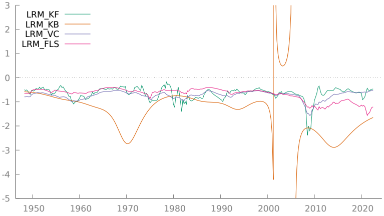
- Roughly, if $u_t \uparrow 1\%$ then $\tilde{y}_t \downarrow 2\%$ according to a rule of thumb
- the LRM (expected ≈ -0.5) is

$$\hat{\kappa}_t = -\frac{\hat{\beta}_{3,t}}{\hat{\beta}_{2,t}},$$

Figure: Long run multipliers

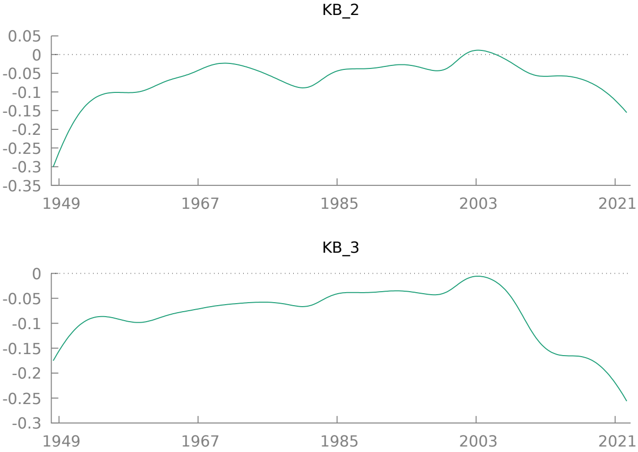


(a) 1948:2–2007:4



(b) 1948:2–2022:3

Figure: KB estimated parameters



The figures reports KB estimators of β_2 (top) and β_3 (bottom).

Table: Computational time (sec)

	1948:2–2007:4	1948:2–2022:3
KF	26.35	45.17
KB	0.009	0.015
VC	387.0	1991
FLS	0.022	0.045

Conclusions

- Compare three estimators for TVP-LMs in `gretl`
- Equivalent in well-behaved data
- Analysis of real data may be challenging
- Computational time and algorithms' stability may differ substantially across alternative choices

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