ARMA-GARCH models

via Kalman filter in Gretl

(Conditional Heteroscedastic State Space Modelling in Gretl)

by

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A Conditional Heteroscedastic State Space Model (CHESS-Model):

$$\mathbf{y_t} = \mathbf{A}_\mathbf{t}^{'}\mathbf{x_t} + \mathbf{H}_\mathbf{t}^{'}\boldsymbol{\xi}_\mathbf{t} + \mathbf{w_t} \qquad \mathbf{w_t} \sim VWN\left(\mathbf{R_t}\right)$$

$$\boldsymbol{\xi}_\mathbf{t+1} = \mathbf{F_t}\,\boldsymbol{\xi}_\mathbf{t} + \mathbf{v_t} \qquad \mathbf{v_t} \sim VWN\left(\mathbf{Q_t}\right)$$

$$\mathbf{R_t} = h_t\mathbf{R}$$

$$\mathbf{Q_t} = h_t\mathbf{Q}$$

$$h_t = \omega + \alpha\mathbf{e'_{t-1}}\,\mathbf{e_{t-1}} + \beta h_{t-1}$$

Example1: Heteroscedastic local level model

$$y_t = \xi_t + w_t \qquad w_t \sim WN(R_t)$$

$$\xi_{t+1} = \xi_t + v_t \qquad v_t \sim WN(Q_t)$$

$$R_t = h_t R$$

$$Q_t = h_t Q$$

fixing $Q = 1$ $\Rightarrow$

$$Q_t = \omega + \alpha e_{t-1}^2 + \beta Q_{t-1}$$

$$R_t = R \cdot Q_t = Q_t / \lambda$$

λ is the signal-noise ratio

Example2: ARMA(1,1)-GARCH(1,1) Model

$$y_t = \mu + \xi_t + \theta_1 \xi_{t-1} \qquad \varepsilon_t \sim WN(h_t)$$

$$\xi_t = \phi_1 \xi_{t-1} + \varepsilon_t \qquad h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$$

$$y_t = \mathbf{A} + \mathbf{H}' \boldsymbol{\xi}_\mathbf{t}$$

$$\boldsymbol{\xi}_\mathbf{t} = \mathbf{F} \boldsymbol{\xi}_\mathbf{t-1} + \mathbf{v}_\mathbf{t}$$

$$\mathbf{Q}_\mathbf{t} = h_t \mathbf{Q}$$

$$\mathbf{A} = [\mu] \qquad \mathbf{H} = \begin{bmatrix} 1 \\ \theta_1 \end{bmatrix} \qquad \boldsymbol{\xi}_\mathbf{t} = \begin{bmatrix} \xi_t \\ \xi_{t-1} \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} \phi_1 & 0 \\ 1 & 0 \end{bmatrix} \qquad \mathbf{v}_\mathbf{t} = \begin{bmatrix} \varepsilon_t \\ 0 \end{bmatrix} \qquad \mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

The Kalman filter in Gretl

- The Kalman filter is a filter that allows to forecast time series from State Space Models
- The Kalman filter is defined in Gretl by a block of commands defining the matrices of the State Space Model:

```
kalman
    obsy y
    obsymat H
    statemat F
    statevar Qt ; modify_q(&Qt, ...)
    ...
end kalman
```

- If a matrix is time-varying, a function modifying that matrix has to be included

The function `modify_q`

```
function void modify_q(matrix *Qt, scalar *ht, matrix Q,
 scalar omega,scalar alpha, scalar beta)

  if $kalman_t=1
      ht = omega /(1-alpha-beta)
  else
      ht = omega + alpha*$kalman_uhat^2 + beta*ht
  endif
  Qt = ht*Q

end function
```

MLE of the model parameters

```
mle ll = ERR ? NA : $kalman_llt
   ...
   Qt = omega/(1-alpha-beta)*Q
   ...
   params ... omega alpha beta
end mle
```

Note It is sufficient to include a convenient, not necessary true, relation between Qt and its parameters, because the right value is recalculated by `modify_q`

A comparative study

1) 300 observations was simulated according to the following ARMA-GARCH model:

$$(1-0.75B)(y_t - 1.2) = (1+0.5B)\varepsilon_t$$

$$\varepsilon_t \sim WN(h_t) \quad h_t = 0.4 + 0.07\varepsilon_{t-1}^2 + 0.85h_{t-1}$$

2) The model was estimated either using the Kalman filter and the conditional log-likelihood

Results

	Kalman filter		*Cond. likelihood*	
param	**estimate**	**sign**	**estimate**	**sign**
mu	1.201	***	1.168	**
phi	0.647	***	0.660	***
theta	0.387	***	0.408	***
omega	0.247		0.423	***
alpha	0.086	**	0.112	***
beta	0.859	***	0.798	***
log.ver	-642.8		-1'306.8	
AIC	1297.5		2'625.5	
BIC	1319.8		2'647.8	

Conclusions

- The Kalman Filter ensures exact maximum likelihood estimates
- Its implementation in Gretl is possible but:
  - in case of non-gaussian disturbances, Kalman filter provides QML estimates;
  - it is quite complex to perform garch(p, q) with p, q > 1
  - the estimate algorithm is quite slow

Thank you
for your attention!