Testing and measuring multi-horizon causality in VAR models:

Two new user-written functions for the Gretl time series menu

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Introduction

- **Both** packages related to the notion of predictability at $h \geq 1$ (multi-horizon causality) - where h denotes horizon
  - The first package DPR (Dufour, Pelletier & Renault, 2006) tests for multi-horizon causality whereas
  - the second package DT2010 (Dufour &Taamouti, 2010) measures causality at $h \geq 1$)
- The standard concept of Granger (1969) causality (GC), $h = 1$, can be generalized to test & measure GC at higher forecasting horizons. Initially raised by Lütkepohl (1993), Lütkepohl & Burda (1997, LB) and Lütkepohl (2005, p. 41-51 and 105-108). Developed theoretically by Dufour & Renault (1998). Empirical testing can be found in LB and DPR
- Economic importance: causal chains or indirect causality can only be revealed at $h > 1$. Auxiliary variables might transmit causality (predictability or indirect dynamic effects)

Multi-horizon causality explained

... as fast as possible ...

- We need a multivariate framework. Let $\underset{m\times 1}{y_{t+1}}$ with $m > 2$, for example: $y_{t+1} = \left(y_{1,t+1}\ ,\ y_{2,t+1}\ ,\ y_{3,t+1}\right)'$
- Consider a trivariate stationary process which follows a $VAR(1)$

$$\begin{bmatrix} y_{1,t+1} \\ y_{2,t+1} \\ y_{3,t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 0.6 & \mathbf{0} & \mathit{0.8} \\ 0 & 0.4 & 0 \\ 0 & \mathit{0.6} & 0.1 \end{bmatrix}}_{\pi} \begin{bmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{bmatrix} + \begin{bmatrix} u_{1,t+1} \\ u_{2,t+1} \\ u_{3,t+1} \end{bmatrix}$$

with u_{t+1} a vector white noise process

- $y_2 \nrightarrow y_1$ at time $t+1$. But this is not the end of the GC story. What happens for $h > 1$;
- Testing GC at $h = 1$ sufficient for (implies) either absence of causal delays or causality at all horizons **only** when $m = 2$

Multi-horizon causality explained

...as fast as possible ...

- At time $t+2$, $y_{t+2} = \pi^2 y_t + \pi u_{t+1} + u_{t+2} = \pi^2 y_t + e_{t+2}$

$$\begin{bmatrix} y_{1,t+2} \\ y_{2,t+2} \\ y_{3,t+2} \end{bmatrix} = \begin{bmatrix} 0.36 & \mathbf{0.48} & 0.56 \\ 0 & 0.16 & 0 \\ 0 & 0.3 & 0.01 \end{bmatrix} \begin{bmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{bmatrix} + \begin{bmatrix} e_{1,t+2} \\ e_{2,t+2} \\ e_{3,t+2} \end{bmatrix}$$

that is $y_2 \overset{h=2}{\rightarrow} y_1$ or in more (revealing) detail

$$y_2 \underset{0.6}{\overset{h=1}{\rightarrow}} y_3 \underset{0.8}{\overset{h=1}{\rightarrow}} y_1 \Rightarrow y_2 \underset{0.48}{\overset{h=2}{\rightarrow}} y_1$$

- Hence causality (predictability) from y_2 to y_1 at horizon $h = 2$ is transmitted through y_3 (indirect causality or causality chain)

Available testing strategies

Nonlinear and linear

Nonlinear (LB)

Estimate $\hat{\pi}$ once from VAR(p)

$H_0 : y_2 \overset{h=1}{\nrightarrow} y_1$ or $[\pi]_{1,2} = 0$,
$stat = g(\hat{\pi})$

$H_0 : y_2 \overset{h=2}{\nrightarrow} y_1$ or $\left[\pi^2\right]_{1,2} = 0$,
$stat = g(\hat{\pi}^2)$

$\vdots$

$H_0 : y_2 \overset{h=H}{\nrightarrow} y_1$ or $\left[\pi^H\right]_{1,2} = 0$,
$stat = g(\hat{\pi}^H)$

- analytical differentiation w.r.t $\hat{\pi}$ algebraically difficult

Linear (DPR) approach

Estimate $\hat{\pi}^{(h)}, h = 1, ..., H$ from (p,h)-autoregressions

$H_0 : y_2 \overset{h=1}{\nrightarrow} y_1$ or $\left[\pi^{(1)}\right]_{1,2} = 0$,
$stat = g(\hat{\pi}^{(1)})$

$H_0 : y_2 \overset{h=2}{\nrightarrow} y_1$ or $\left[\pi^{(2)}\right]_{1,2} = 0$,
$stat = g(\hat{\pi}^{(2)})$

$\vdots$

$H_0 : y_2 \overset{h=H}{\nrightarrow} y_1$ or $\left[\pi^{(H)}\right]_{1,2} = 0$,
$stat = g(\hat{\pi}^{(H)})$

The Dufour, Pelletier & Renault (2006, DPR) method

- Based on $(p,h)-$autoregresssions

$$
\begin{array}{l|l}
Y_{t+h} = \mu_t^{(h)} + \sum_{k=1}^{p} \pi_k^{(h)} Y_{t+1-k} + \sum_{k=p+1}^{d} \pi_k^{(h)} Y_{t+1-k} + e_{t+h}^{(h)} & E\left(u_t u_t'\right) = \Omega \\
\hline
e_{t+h}^{(h)} = \sum_{j=0}^{h-1} \psi_h u_{t+h-j} & \psi_h = \pi_1^{(h)}
\end{array}
$$

Figure out (in advance) p, d and $\mu_t^{(h)}$. Typically $\mu_t^{(h)} = \mu$. Package allows for linear and quadratic trends

1. OLS estimation of $\hat{\Pi}^{(h)} = \left(\pi_1^{(h)}, ..., \pi_p^{(h)}\right)'$ (leave $\pi_{p+1}^{(h)}, .., \pi_{p+d}^{(h)}$ out of testing procedure).Keep column $\hat{\Pi}_i^{(h)}$ from i^{th} equation
2. Wald stat: $W^{(h)} = T\left(R\hat{\Pi}_i^{(h)}\right)'\left(R\hat{V}^{NW}R'\right)^{-1}\left(R\hat{\Pi}_i^{(h)}\right) \xrightarrow{d} \chi_p^2$
3. $\hat{V}^{NW}$ Newey and West (1987) HAC variance-covariance estimator, truncation $m_{trunc} = h - 1$

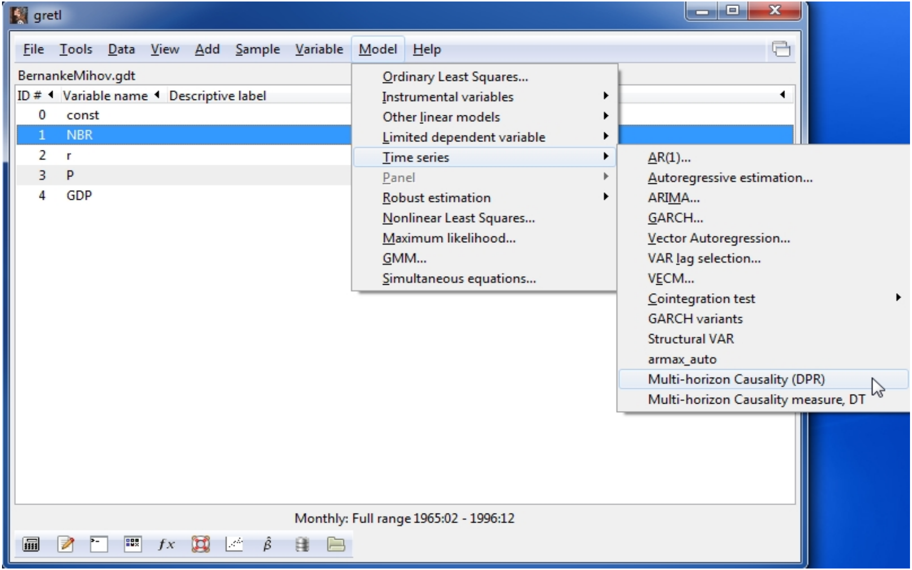
The Dufour, Pelletier & Renault (2006, DPR) method

- Based on $(p,h)-$autoregresssions

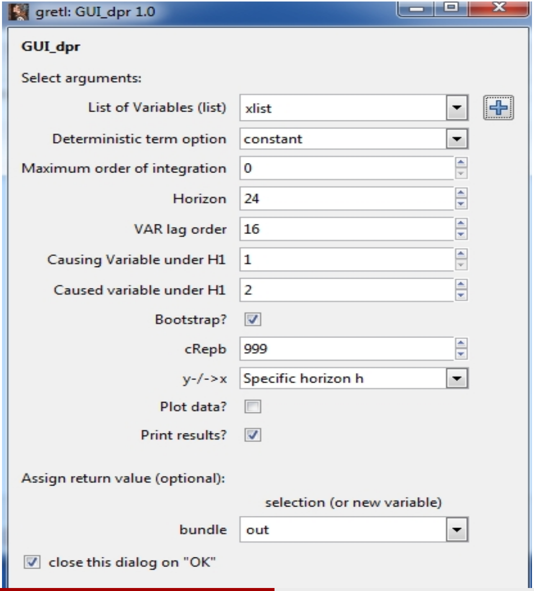
$Y_{t+h} = \mu_t^{(h)} + \sum_{k=1}^{p} \pi_k^{(h)} Y_{t+1-k} + \sum_{k=p+1}^{d} \pi_k^{(h)} Y_{t+1-k} + e_{t+h}^{(h)}$	$E\left(u_t u_t'\right) = \Omega$
$e_{t+h}^{(h)} = \sum_{j=0}^{h-1} \psi_h u_{t+h-j}$	$\psi_h = \pi_1^{(h)}$

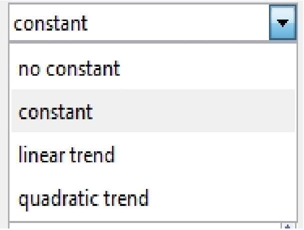
4. N simulated samples using $\hat{\Pi}^{(h)}$, $\hat{\psi}_h = \hat{\pi}_1^{(h)}$, $\hat{\Omega}$ and $u_t \sim N.i.d\left(0, \hat{\Omega}\right)$. Impose non-causality constraints and calculate $W_n^{(h)}$, $1 \leq n \leq N$
5. Simulated p-value: $\hat{p}_N = \frac{1}{N+1}\left(1 + \sum_{n=1}^{N} \mathbf{1}\left\{W_n^{(h)} - W^{(h)}\right\}\right)$. Reject non-causality at horizon h and significance level α if $\hat{p}_N \leq \alpha$

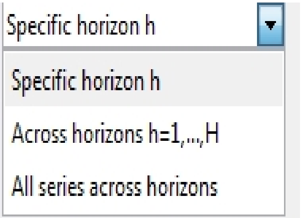
DPR package in Gretl



DPR package in Gretl: GUI

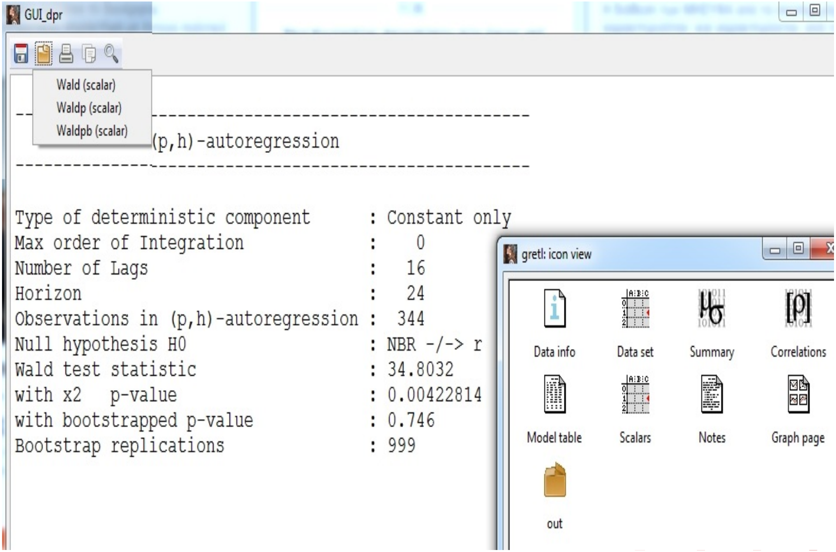






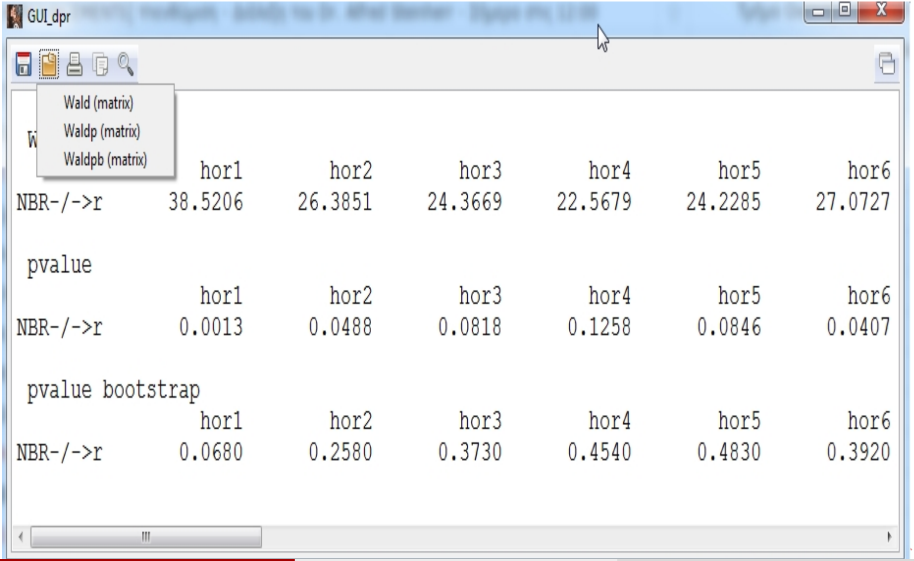
DPR package in Gretl.

Output: specific horizon *h*



DPR package in Gretl.

Output: across horizons $h = 1, \ldots, H$



W

	hor1	hor2	hor3	hor4	hor5	hor6
NBR-/->r	38.5206	26.3851	24.3669	22.5679	24.2285	27.0727

pvalue

	hor1	hor2	hor3	hor4	hor5	hor6
NBR-/->r	0.0013	0.0488	0.0818	0.1258	0.0846	0.0407

pvalue bootstrap

	hor1	hor2	hor3	hor4	hor5	hor6
NBR-/->r	0.0680	0.2580	0.3730	0.4540	0.4830	0.3920

DPR package in Gretl.

Output: All series across horizons

GUI_dpr

Wald (matrix)
Waldp (matrix)
Waldpb (matrix)

Nu ...tions: 10

Wald

	hor1	hor2	hor3	hor4	hor5	hor6	hor7	hc
NBR-/->r	38.5206	26.3851	24.3669	22.5679	24.2285	27.0727	21.4339	17.21
NBR-/->P	50.5512	45.2468	49.1373	33.8523	33.8915	35.3909	39.1761	40.01
NBR-/->GDP	27.8031	25.0114	25.5109	23.6785	15.0037	17.5746	16.6777	19.66
r-/->NBR	22.5395	20.8631	17.2362	17.4229	17.8954	18.6471	25.2065	40.99
r-/->P	32.2534	32.9285	32.0432	25.0201	25.2527	25.8177	29.2622	29.60
r-/->GDP	26.3351	28.6642	35.9767	38.8678	37.6775	39.8131	64.1479	80.23
P-/->NBR	17.0670	18.9469	16.6844	21.2361	28.7231	20.0357	18.2290	23.66
P-/->r	22.4393	16.4460	14.3076	14.2927	14.0141	16.7135	11.5584	16.57
P-/->GDP	20.6929	24.1138	27.4125	23.3593	22.9122	18.5586	20.8223	23.69
GDP-/->NBR	17.6338	20.6570	24.5341	18.3230	18.3132	32.8748	33.9974	39.66
GDP-/->r	42.5322	43.0068	49.8347	41.1073	41.7934	36.0957	29.5462	25.78
GDP-/->P	24.3375	24.4941	24.6136	22.8175	26.7914	40.0846	36.4887	49.61

pvalue

	hor1	hor2	hor3	hor4	hor5	hor6	hor7	hc
NBR-/->r	0.0013	0.0488	0.0818	0.1258	0.0846	0.0407	0.1624	0.37
NBR-/->P	0.0000	0.0001	0.0000	0.0057	0.0056	0.0035	0.0010	0.00
NBR-/->GDP	0.0334	0.0696	0.0613	0.0968	0.5244	0.3494	0.4067	0.23
r-/->NBR	0.1266	0.1838	0.3705	0.3588	0.3301	0.2874	0.0663	0.00
r-/->P	0.0093	0.0076	0.0099	0.0695	0.0655	0.0566	0.0222	0.02
r-/->GDP	0.0495	0.0263	0.0029	0.0011	0.0017	0.0008	0.0000	0.00
P-/->NBR	0.3813	0.2714	0.4063	0.1696	0.0259	0.2186	0.3107	0.09
P-/->r	0.1296	0.4223	0.5758	0.5769	0.5977	0.4044	0.7738	0.41
P-/->GDP	0.1906	0.0870	0.0371	0.1045	0.1161	0.2922	0.1854	0.09
GDP-/->NBR	0.3458	0.1921	0.0785	0.3053	0.3059	0.0077	0.0054	0.00
GDP-/->r	0.0003	0.0003	0.0000	0.0005	0.0004	0.0028	0.0205	0.05
GDP-/->P	0.0824	0.0793	0.0769	0.1187	0.0439	0.0008	0.0025	0.00

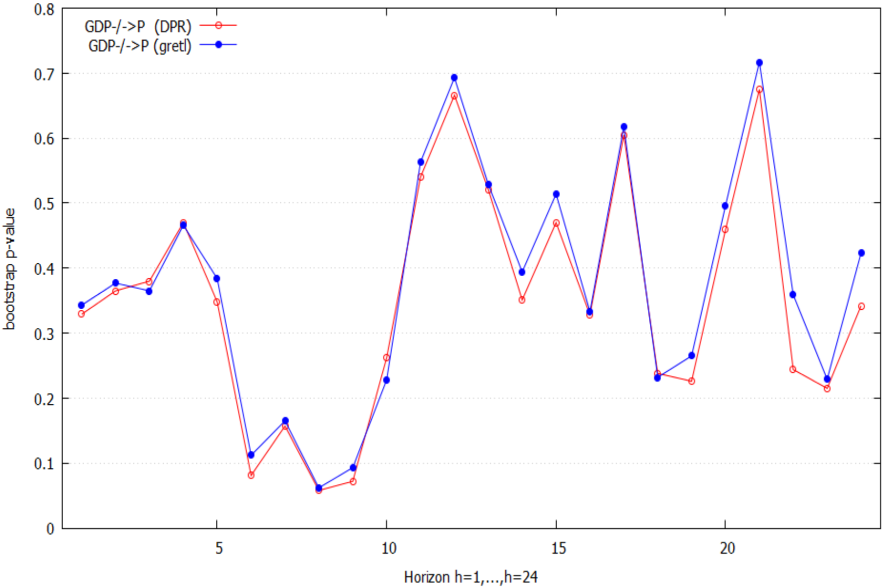
pvalue bootstrap

	hor1	hor2	hor3	hor4	hor5	hor6	hor7	hc
NBR-/->r	0.0660	0.2730	0.3880	0.4650	0.4600	0.3940	0.6280	0.82
NBR-/->P	0.0040	0.0310	0.0240	0.1860	0.1890	0.1940	0.1650	0.19
NBR-/->GDP	0.2640	0.3700	0.3960	0.4680	0.8590	0.8000	0.8540	0.78
r-/->NBR	0.4670	0.5720	0.7500	0.7750	0.7840	0.7740	0.5700	0.17
r-/->P	0.1410	0.1340	0.1840	0.4210	0.4600	0.4590	0.3900	0.41

DPR package in Gretl. Replication Results ✓

DPR										
h:	hor1	hor2	hor3	hor4	hor5	...	hor21	hor22	hor23	hor24
NBR-/->r	38.5205	26.3851	24.3672	22.5684	24.2294	...	66.1538	38.7272	28.8194	34.8015
NBR-/->P	50.5547	45.2498	49.1408	33.8545	33.8943	...	55.0460	64.8568	54.8929	42.2509
NBR-/->GDP	27.8029	25.0122	25.5123	23.6799	15.0040	...	39.0652	43.6661	56.9724	67.7764
r-/->NBR	22.5390	20.8621	17.2357	17.4222	17.8944	...	37.7470	51.4196	33.2897	27.3362
r-/->P	32.2481	32.9207	32.0362	25.0124	25.2441	...	57.1530	54.6753	69.6377	59.2184
r-/->GDP	26.3362	28.6645	35.9768	38.8680	37.6788	...	65.5025	71.3334	63.1942	64.8020
P-/->NBR	17.0696	18.9504	16.6880	21.2381	28.7264	...	50.2743	45.8078	54.1961	56.7865
P-/->r	22.4385	16.4455	14.3073	14.2932	14.0148	...	16.9652	31.9336	28.8377	30.0013
P-/->GDP	20.6903	24.1099	27.4106	23.3585	22.9095	...	66.5988	59.2137	71.5165	67.1851
GDP-/->NBR	17.6338	20.6568	24.5334	18.3220	18.3123	...	40.7051	36.3960	38.7162	40.8466
GDP-/->r	42.5327	43.0078	49.8366	41.1082	41.7945	...	38.8175	38.7110	38.9987	25.0577
GDP-/->P	24.3368	24.4925	24.6125	22.8160	26.7900	...	31.6598	54.0684	65.8284	53.0823
gretl										
h:	hor1	hor2	hor3	hor4	hor5	...	hor21	hor22	hor23	hor24
NBR-/->r	38.5206	26.3851	24.3669	22.5679	24.2285	...	66.1543	38.7290	28.8213	34.8032
NBR-/->P	50.5512	45.2468	49.1373	33.8523	33.8915	...	55.0428	64.8494	54.8913	42.2488
NBR-/->GDP	27.8031	25.0114	25.5109	23.6785	15.0037	...	39.0670	43.6679	56.9772	67.7810
r-/->NBR	22.5395	20.8631	17.2362	17.4229	17.8954	...	37.7416	51.4164	33.2892	27.3350
r-/->P	32.2534	32.9285	32.0432	25.0201	25.2527	...	57.1504	54.6704	69.6278	59.2099
r-/->GDP	26.3351	28.6642	35.9767	38.8678	37.6775	...	65.5041	71.3383	63.1989	64.8086
P-/->NBR	17.0670	18.9469	16.6844	21.2361	28.7231	...	50.2713	45.8035	54.1886	56.7819
P-/->r	22.4393	16.4460	14.3076	14.2927	14.0141	...	16.9652	31.9379	28.8408	30.0024
P-/->GDP	20.6929	24.1138	27.4125	23.3593	22.9122	...	66.6108	59.2259	71.5332	67.2017
GDP-/->NBR	17.6338	20.6570	24.5341	18.3230	18.3132	...	40.7055	36.3952	38.7162	40.8469
GDP-/->r	42.5322	43.0068	49.8347	41.1073	41.7934	...	38.8169	38.7133	39.0002	25.0563
GDP-/->P	24.3375	24.4941	24.6136	22.8175	26.7914	...	31.6593	54.0694	65.8381	53.0882

DPR package in Gretl. Replication Results ✓



Causality measures. The Dufour & Taamouti (2010, DT) procedure

- Let $Y_t = (y_{1,t}\ , y_{2,t}\ , Y_{3,t})'$ the available detrended series where $y_{1,t}, y_{2t}$ are 1×1 and $Y_{3,t}$ is $m_3 \times 1$. Based on two VAR estimations, unrestricted and restricted with $Y_{0,t} = (y_{1,t}\ , Y_{3,t})'$ (drop the second variable from available information set)

$$Y_t = \sum_{k=1}^{p} \pi_k Y_{t-k} + u_t \quad \Big| \quad Y_{0,t} = \sum_{k=1}^{p} \pi_{0,k} Y_{0,t-k} + e_t$$

1. Figure out p prior to building the causality measures. Keep $\hat{u}_t$, $\hat{\pi}_k$, $k = 1, ..., p$ and the variance-covariance matrix $\hat{\Sigma}_{u|p}$ of the error term. Compute the $VMA(\infty)$ $\hat{\Psi}_j$, $j = 0, ..., h-1$, $\hat{\Psi}_0 = I_K$, coefficient matrices by recursive substitution based on $\hat{\pi}_k$ and the unconstrained forecast error variance-covariance matrix $\hat{\Sigma}_p(h) = \sum_{j=0}^{h-1} \hat{\Psi}_j \hat{\Sigma}_{u|p} \hat{\Psi}_j'$ at horizon h.

   Similarly, for the constrained model calculate $\tilde{\Sigma}_{0|p}(h) = \sum_{j=0}^{h-1} \tilde{\Psi}_j \tilde{\Sigma}_{e|p} \tilde{\Psi}_j'$

Causality measures. The Dufour & Taamouti (2010, DT) procedure

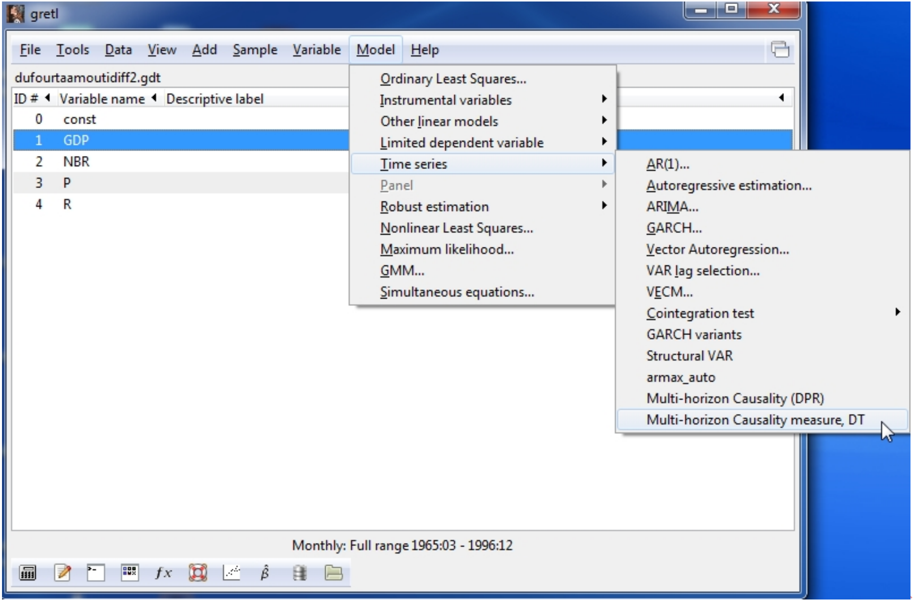
2. Causality measure (C.M) estimate is given by

$$\hat{C}_L(h) = \hat{C}_L\left(y_2 \underset{h}{\rightarrow} y_1\right) = \ln\left(\frac{\left[\tilde{\Sigma}_{0|p}(h)\right]_{1,1}}{\left[\hat{\Sigma}_p(h)\right]_{1,1}}\right)$$

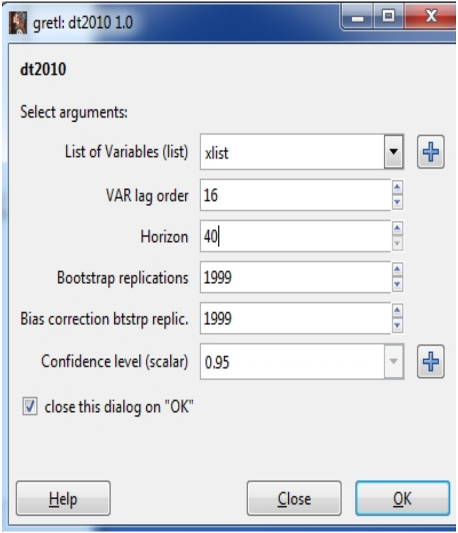
4. Residual-based bootstrap method proposed by DT to compute the confidence interval (lower bound, L.B & upper bound, U.B) of $\hat{C}_L(h)$ at a given horizon h. A mean bias correction is applied and a non-negativity truncation is imposed

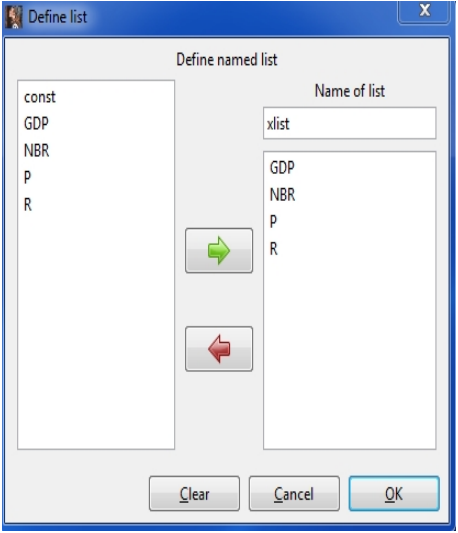
3. Prior to bootstraping C.M, a separate bootstrap using $\hat{u}_t$, $\hat{\pi}_k$ is employed to correct the bias in the estimated VAR coefficients. In step 4, the bias corrected coefficients are employed

DT package in Gretl

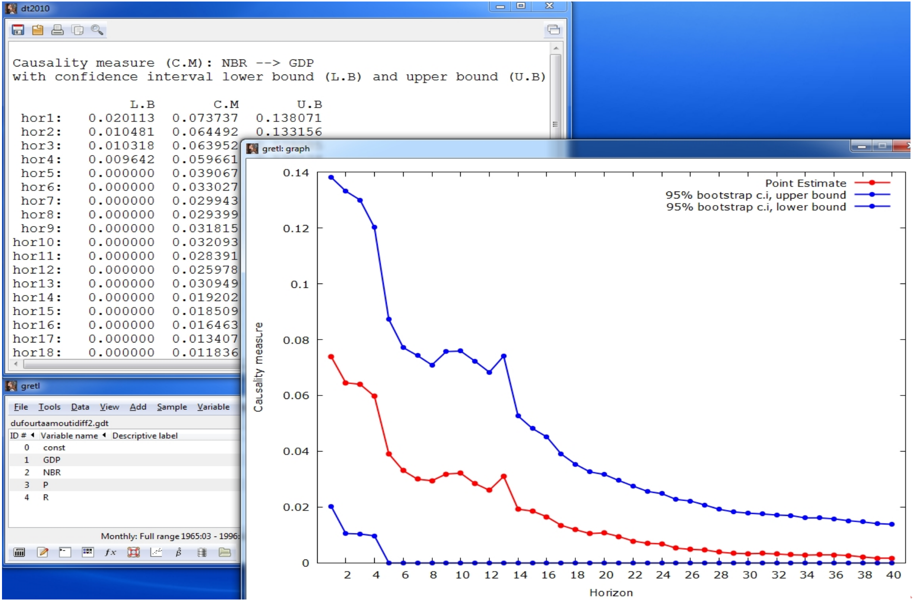


DT package in Gretl:GUI

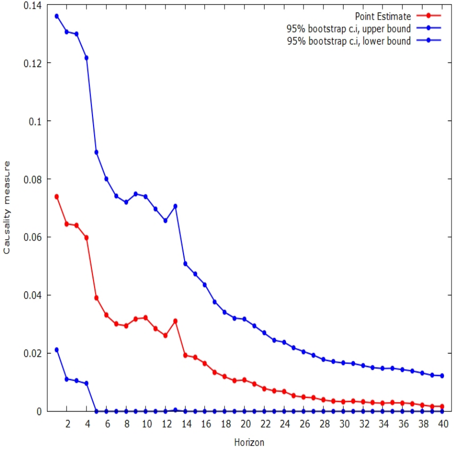


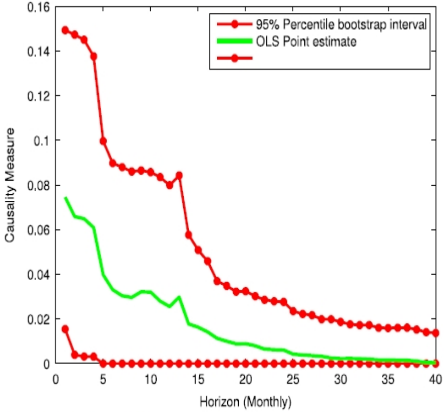


DT package in Gretl: output



DT package in Gretl. Replication results





(c) Causality measures from nonborrowed reserves to real GDP.

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