Testing and measuring multi-horizon causality in VAR models:

Two new user-written functions for the Gretl time series menu

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multi-horizon causality

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Introduction

- Both packages related to the notion of predictability at *h* ≥ 1 (multi-horizon causality) - where *h* denotes horizon
 - The first package DPR (Dufour, Pelletier & Renault, 2006) tests for multi-horizon causality whereas
 - the second package DT2010 (Dufour &Taamouti, 2010) measures causality at *h* ≥ 1)
- The standard concept of Granger (1969) causality (GC), h = 1, can be generalized to test & measure GC at higher forecasting horizons. Initially raised by Lütkepohl (1993), Lütkepohl & Burda (1997, LB) and Lütkepohl (2005, p. 41-51 and 105-108). Developed theoretically by Dufour & Renault (1998). Empirical testing can be found in LB and DPR
- Economic importance: causal chains or indirect causality can only be revealed at h > 1. Auxiliary variables might transmit causality (predictability or indirect dynamic effects)

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Multi-horizon causality explained

... as fast as possible ...

• We need a multivariate framework. Let y_{t+1} with m > 2, for example: $y_{t+1} = (y_{1,t+1}, y_{2,t+1}, y_{3,t+1})'$

• Consider a trivariate stationary process which follows a VAR(1)

$$\begin{bmatrix} y_{1,t+1} \\ y_{2,t+1} \\ y_{3,t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 0.6 & \mathbf{0} & 0.8 \\ 0 & 0.4 & 0 \\ 0 & 0.6 & 0.1 \end{bmatrix}}_{\pi} \begin{bmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{bmatrix} + \begin{bmatrix} u_{1,t+1} \\ u_{2,t+1} \\ u_{3,t+1} \end{bmatrix}$$

with u_{t+1} a vector white noise process

- y₂ → y₁ at time t + 1. But this is not the end of the GC story. What happens for h > 1;
- Testing GC at h = 1 sufficient for (implies) either absence of causal delays or causality at all horizons only when m = 2

Multi-horizon causality explained

... as fast as possible ...

• At time
$$t + 2$$
, $y_{t+2} = \pi^2 y_t + \pi u_{t+1} + u_{t+2} = \pi^2 y_t + e_{t+2}$

[<i>y</i> _{1,<i>t</i>+2}]		0.36	0.48	0.56	[y _{1,t}]		[<i>e</i> _{1,<i>t</i>+2}]
<i>Y</i> _{2,<i>t</i>+2}	=	0	0.16	0	y _{2,t}	+	<i>e</i> _{2,<i>t</i>+2}
<i>y</i> _{3,t+2}		L O	0.3	0.01	y 3,t		[<i>e</i> _{3,t+2}]

that is $y_2 \stackrel{h=2}{\rightarrow} y_1$ or in more (revealing) detail

$$y_2 \stackrel{h=1}{\underset{0.6}{\rightarrow}} y_3 \stackrel{h=1}{\underset{0.8}{\rightarrow}} y_1 \Rightarrow y_2 \stackrel{h=2}{\underset{0.48}{\rightarrow}} y_1$$

 Hence causality (predictability) from y₂ to y₁ at horizon h = 2 is transmitted through y₃ (indirect causality or causality chain)

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Available testing strategies

Nonlinear and linear

Nonlinear (LB)

Estimate $\hat{\pi}$ once from VAR(p) $H_0: y_2 \stackrel{h=1}{\nrightarrow} y_1 \text{ or } [\pi]_{1,2} = 0,$ stat = $g(\hat{\pi})$ $H_0: y_2 \stackrel{h=2}{\nrightarrow} y_1 \text{ or } [\pi^2]_{1,2} = 0 ,$ stat = $g(\hat{\pi}^2)$ $H_0: y_2 \stackrel{h=H}{\nrightarrow} y_1 \text{ or } \left[\pi^H\right]_{1,2} = 0$, stat = $q(\hat{\pi}^H)$ \blacktriangleright analytical differentiation w.r.t $\hat{\pi}$ algebraically difficult

Linear (DPR) approach

Estimate $\hat{\pi}^{(h)}, h = 1, ..., H$ from (p,h)-autoregressions $H_0: y_2 \xrightarrow{h=1}{\to} y_1 \text{ or } [\pi^{(1)}]_{1,2} = 0,$ stat = $g(\hat{\pi}^{(1)})$ $H_0: y_2 \xrightarrow{h=2}{\to} y_1 \text{ or } [\pi^{(2)}]_{1,2} = 0,$ stat = $q(\hat{\pi}^{(2)})$ $H_0: y_2 \xrightarrow{h=H} y_1 \text{ or } \left[\pi^{(H)}\right]_{1,2} = 0$, stat = $q(\hat{\pi}^{(H)})$

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The Dufour, Pelletier & Renault (2006, DPR) method

• Based on (*p*, *h*)-autoregresssions

$$\begin{aligned} \mathbf{Y}_{t+h} &= \mu_t^{(h)} + \sum_{k=1}^p \pi_k^{(h)} \mathbf{Y}_{t+1-k} + \sum_{k=p+1}^d \pi_k^{(h)} \mathbf{Y}_{t+1-k} + \mathbf{e}_{t+h}^{(h)} \quad \mathbf{E} \left(u_t u_t' \right) = \Omega \\ \mathbf{e}_{t+h}^{(h)} &= \sum_{j=0}^{h-1} \psi_h u_{t+h-j} \quad \psi_h = \pi_1^{(h)} \end{aligned}$$

Figure out (in advance) p, d and $\mu_t^{(h)}$. Typically $\mu_t^{(h)} = \mu$. Package allows for linear and quadratic trends

- 1. OLS estimation of $\hat{\Pi}^{(h)} = (\pi_1^{(h)}, ..., \pi_p^{(h)})'$ (leave $\pi_{p+1}^{(h)}, ..., \pi_{p+d}^{(h)}$ out of testing procedure). Keep column $\hat{\Pi}_i^{(h)}$ from *i*th equation
- 2. Wald stat: $W^{(h)} = T\left(R\hat{\Pi}_{i}^{(h)}\right)'\left(R\hat{V}^{NW}R'\right)^{-1}\left(R\hat{\Pi}_{i}^{(h)}\right) \xrightarrow{d} \chi_{p}^{2}$
- 3. \hat{V}^{NW} Newey and West (1987) HAC variance-covariance estimator, truncation $m_{trunc} = h 1$

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The Dufour, Pelletier & Renault (2006, DPR) method

Based on (p, h)-autoregressions

- 4. *N* simulated samples using $\hat{\Pi}^{(h)}$, $\hat{\psi}_h = \hat{\pi}_1^{(h)}$, $\hat{\Omega}$ and $u_t \sim N.i.d(0, \hat{\Omega})$. Impose non-causality constraints and calculate $W_n^{(h)}$, $1 \le n \le N$
- 5. Simulated p-value: $\hat{p}_N = \frac{1}{N+1} \left(1 + \sum_{n=1}^N \mathbf{1} \left\{ W_n^{(h)} W^{(h)} \right\} \right)$. Reject non-causality at horizon *h* and significance level α if $\hat{p}_N \leq \alpha$

DPR package in Gretl

Eile Iools Data View Add Sample Variable Model Help BernankeMihov.gdt Ordinary Least Squares ID # 4 Variable name 4 Descriptive label Instrumental variables 0 const Other finear models 1 NBR Limited dependent variable 2 r Imme series 3 P Banel 4 GDP Banel Maximum likelihood GARCH Yector Autoregression VAR Jag selection VAR Jag selection Variants Simultaneous equations Contegration test Multi-horizon Causality (DPR) Multi-horizon Causality measure, DT
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3 P Panel Autoregressive estimation 4 GDP Bobust estimation ARIMA Vonlinear Least Squares GARCH GARCH Maximum likelihood GMM Vector Autoregression Simultaneous equations VECM Cointegration test GARCH variants Structural VAR armax_auto Multi-horizon Causality (DPR) Multi-horizon Causality measure, DT
4 GDP Robust estimation ARIMA ARIMA Solution Robust estimation Nonlinear Least Squares Maximum likelihood GMM GMM Simultaneous equations VAR Jag selection Cointegration test GARCH variants Structural VAR armax_auto Multi-horizon Causality (DPR) Multi-horizon Causality (DPR)
Nonlinear Least Squares GARCH Maximum likelihood Vector Autoregression GMM VAR Jag selection Simultaneous equations VECM Cointegration test + GARCH variants Structural VAR armax_auto Multi-horizon Causality (DPR) Multi-horizon Causality measure, DT -
Maximum likelihood Yector Autoregression GMM VA Jag selection Simultaneous equations VECM Contegration test GARCH variants Structural VAR armax_auto Multi-horizon Causality (DPR) Multi-horizon Causality measure, DT
GMM VAR lag selection Simultaneous equations VECM Contegration test GARCH variants Structural VAR armax_auto Multi-horizon Causality (DPR) Multi-horizon Causality measure, DT
Simultaneous equations VECM Cointegration test GARCH variants Structural VAR armax_auto Multi-horizon Causality (DPR) Multi-horizon Causality measure, DT
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Multi-horizon Causality measure, DT
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DPR package in Gretl: GUI

gretl: GUI_dpr 1.0	
GUI_dpr	
Select arguments:	
List of Variables (list)	xlist 💌 🛃
Deterministic term option	constant 💌
Maximum order of integration	0
Horizon	24
VAR lag order	16
Causing Variable under H1	1
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Bootstrap?	
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y-/->x	Specific horizon h
Plot data?	
Print results?	V
Assign return value (optional):	
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✓ close this dialog on "OK"	
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constant	-
no constant	
constant	
linear trend	
quadratic trend	
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Specific horizon h	•	
Specific horizon h		
Across horizons h=1,,H		
All series across horizons		
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DPR package in Gretl.

Output: specific horizon h

Wald(scalar) Waldp(scalar) Waldpb(scalar) (p,h) -autoregression					
Type of deterministic component Max order of Integration Number of Lags Horizon Observations in (p,h)-autoregression Null hypothesis H0 Wald test statistic with x2 p-value with bootstrapped p-value Bootstrap replications	: Constant onl : 0 : 16 : 24 : 344 : NER -/-> r : 34.8032 : 0.00422814 : 0.746 : 999	y grett: icon view Data info Model table imitian out	Data set	Kummary Bir Notes	Correlations Graph page

DPR package in Gretl.

Output: across horizons h = 1, ..., H

GUI_dpr	cupitral - Post	(to D. After S	teriter - Dyspa		1 1/10	
				h3		0
Wald (matrix) Waldp (matrix)						
Waldpb (matrix)	hor1	hor2	hor3	hor4	hor5	hor6
NBR-/->r	38.5206	26.3851	24.3669	22.5679	24.2285	27.0727
pvalue						
-	hor1	hor2	hor3	hor4	hor5	hor6
NBR-/->r	0.0013	0.0488	0.0818	0.1258	0.0846	0.0407
pvalue boots	strap					
-	hor1	hor2	hor3	hor4	hor5	hor6
NBR-/->r	0.0680	0.2580	0.3730	0.4540	0.4830	0.3920
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DPR package in Gretl.

Output: All series across horizons

GUL_dpr	the second s	-	-					
BBBCQ								
Wald (matrix)								
Maldp (matrix)	tions	1.0						
Waldpb (matrix)	Cromb.	10						
	La							
Wald								
	hor1	hor2	hor3	hor4	hor5	horf	hor7	hc
NBB-/->r	38,5206	26.3851	24.3669	22.5679	24.2285	27.0727	21,4339	17.21
NBR-/->P	50.5512	45.2468	49,1373	33.8523	33.8915	35,3909	39,1761	40.01
NBR-/->CDP	27 8031	25 0114	25 5109	23 6785	15 0037	17 5746	16 6777	19 66
r-/->NBR	22 5395	20 8631	17 2362	17 4229	17 8954	18 6471	25 2065	40.99
r-/->P	32 2534	32 9285	32 0432	25 0201	25 2527	25 8177	29 2622	29 60
r-/->GDP	26 3351	28 6642	35 9767	38 8678	37 6775	39 8131	64 1479	80 23
D-/->NPP	17 0670	18 9/69	16 6944	21 2361	28 7231	20 0357	18 2290	23 66
P-/->r	22 /393	16 4460	14 3076	14 2927	14 0141	16 7135	11 5594	16 55
P-/->CDP	20 6929	24 1139	27 4125	23 3593	22 0122	18 5586	20 8223	23 60
CDP-/->NPP	17 6338	20.6570	24 5341	18 3230	18 3132	32 8748	33 9974	39 66
CDP-/->r	42 5322	43 0068	49 8347	41 1073	11 7934	36 0957	29 5462	25 76
GDF / >I	24 2275	24 4041	24 6126	22 0175	26 7014	40.0946	26 4997	40 61
GDF-/-/F	24.3375	24.4941	24.0130	22.01/5	20.7914	40.0040	30.4007	49.01
pyalue								
pvarue	hor1	hor?	hor3	hor/	bor5	borf	hor7	hc
NED- (->x	0 0012	0 0499	0 0919	0 1259	0 0946	0 0407	0 1624	0 35
NDR-/->L	0.0013	0.0400	0.0010	0.1250	0.0046	0.0407	0.1024	0.37
NDR-/->P	0.0000	0.0001	0.0000	0.0057	0.5344	0.0035	0.0010	0.00
NDR-/->GDF	0.0334	0.1030	0.0015	0.0900	0.3244	0.3494	0.4007	0.20
1-/->NBR	0.1200	0.1030	0.0000	0.0500	0.0655	0.2074	0.0003	0.00
1-/->F	0.0093	0.0070	0.0099	0.0095	0.0000	0.0000	0.0222	0.02
I-/->GDP	0.0495	0.0263	0.0029	0.0011	0.0017	0.0008	0.0000	0.00
P-/->NBR	0.3013	0.2714	0.4063	0.1696	0.0259	0.2100	0.3107	0.03
	0.1296	0.4223	0.5758	0.5769	0.5977	0.4044	0.7730	0.41
CDD (>NDD	0.1906	0.0870	0.0371	0.1045	0.1161	0.2922	0.1054	0.05
GDP-/->NBR	0.3458	0.1921	0.0785	0.3053	0.3059	0.0077	0.0054	0.00
GDP-/->r	0.0003	0.0003	0.0000	0.0005	0.0004	0.0028	0.0205	0.05
GDP-/->P	0.0824	0.0793	0.0769	0.1187	0.0439	0.0008	0.0025	0.00
muslue heat	atuon							
pvarue boot	SLIAP	hom?	hom?	how	howE	howf	hom7	he
NDD (> m	0 0660	0 2720	0 3000	0 4650	0 4600	0 3040	0 6280	110
NDR-/->r	0.0660	0.2730	0.3880	0.4650	0.4600	0.3940	0.0280	0.82
NBR-/->P	0.0040	0.0310	0.0240	0.1860	0.1890	0.1940	0.1650	0.15
NBR-/->GDP	0.2640	0.3700	0.3960	0.4680	0.8590	0.8000	0.8540	0.78
r-/->NBR	0.4670	0.5720	0.7500	0.7750	0.7840	0.//40	0.5700	0.15
r=/=>P	0 1410	0 1340	0 19/0	0 4910	0 4600	0 /500	1 30NU	

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DPR package in Gretl. Replication Results √

DPR										
h:	hor1	hor2	hor3	hor4	hor5		hor21	hor22	hor23	hor24
NBR-/->r	38.5205	26.3851	24.3672	22.5684	24.2294		66.1538	38.7272	28.8194	34.8015
NBR-/->P	50.5547	45.2498	49.1408	33.8545	33.8943		55.0460	64.8568	54.8929	42.2509
NBR-/->GDP	27.8029	25.0122	25.5123	23.6799	15.0040		39.0652	43.6661	56.9724	67.7764
r-/->NBR	22.5390	20.8621	17.2357	17.4222	17.8944		37.7470	51.4196	33.2897	27.3362
r-/->P	32.2481	32.9207	32.0362	25.0124	25.2441		57.1530	54.6753	69.6377	59.2184
r-/->GDP	26.3362	28.6645	35.9768	38.8680	37.6788		65.5025	71.3334	63.1942	64.8020
P-/->NBR	17.0696	18.9504	16.6880	21.2381	28.7264		50.2743	45.8078	54.1961	56.7865
P-/->r	22.4385	16.4455	14.3073	14.2932	14.0148		16.9652	31.9336	28.8377	30.0013
P-/->GDP	20.6903	24.1099	27.4106	23.3585	22.9095		66.5988	59.2137	71.5165	67.1851
GDP-/->NBR	17.6338	20.6568	24.5334	18.3220	18.3123		40.7051	36.3960	38.7162	40.8466
GDP-/->r	42.5327	43.0078	49.8366	41.1082	41.7945		38.8175	38.7110	38.9987	25.0577
GDP-/->P	24.3368	24.4925	24.6125	22.8160	26.7900		31.6598	54.0684	65.8284	53.0823
gretl										
gretl h:	hor1	hor2	hor3	hor4	hor5		hor21	hor22	hor23	hor24
gretl h: NBR-/->r	hor1 38.5206	hor2 26.3851	hor3 24.3669	hor4 22.5679	hor5 24.2285		hor21 66.1543	hor22 38.7290	hor23 28.8213	hor24 34.8032
<mark>gretl</mark> h: NBR-/->r NBR-/->P	hor1 38.5206 50.5512	hor2 26.3851 45.2468	hor3 24.3669 49.1373	hor4 22.5679 33.8523	hor5 24.2285 33.8915	 	hor21 66.1543 55.0428	hor22 38.7290 64.8494	hor23 28.8213 54.8913	hor24 34.8032 42.2488
gretl h: NBR-/->r NBR-/->P NBR-/->GDP	hor1 38.5206 50.5512 27.8031	hor2 26.3851 45.2468 25.0114	hor3 24.3669 49.1373 25.5109	hor4 22.5679 33.8523 23.6785	hor5 24.2285 33.8915 15.0037	 	hor21 66.1543 55.0428 39.0670	hor22 38.7290 64.8494 43.6679	hor23 28.8213 54.8913 56.9772	hor24 34.8032 42.2488 67.7810
gretl h: NBR-/->r NBR-/->P NBR-/->GDP r-/->NBR	hor1 38.5206 50.5512 27.8031 22.5395	hor2 26.3851 45.2468 25.0114 20.8631	hor3 24.3669 49.1373 25.5109 17.2362	hor4 22.5679 33.8523 23.6785 17.4229	hor5 24.2285 33.8915 15.0037 17.8954	 	hor21 66.1543 55.0428 39.0670 37.7416	hor22 38.7290 64.8494 43.6679 51.4164	hor23 28.8213 54.8913 56.9772 33.2892	hor24 34.8032 42.2488 67.7810 27.3350
gret1 h: NBR-/->r NBR-/->P NBR-/->GDP r-/->NBR r-/->P	hor1 38.5206 50.5512 27.8031 22.5395 32.2534	hor2 26.3851 45.2468 25.0114 20.8631 32.9285	hor3 24.3669 49.1373 25.5109 17.2362 32.0432	hor4 22.5679 33.8523 23.6785 17.4229 25.0201	hor5 24.2285 33.8915 15.0037 17.8954 25.2527	 	hor21 66.1543 55.0428 39.0670 37.7416 57.1504	hor22 38.7290 64.8494 43.6679 51.4164 54.6704	hor23 28.8213 54.8913 56.9772 33.2892 69.6278	hor24 34.8032 42.2488 67.7810 27.3350 59.2099
gretl h: NBR-/->r NBR-/->P NBR-/->GDP r-/->NBR r-/->P r-/->GDP	hor1 38.5206 50.5512 27.8031 22.5395 32.2534 26.3351	hor2 26.3851 45.2468 25.0114 20.8631 32.9285 28.6642	hor3 24.3669 49.1373 25.5109 17.2362 32.0432 35.9767	hor4 22.5679 33.8523 23.6785 17.4229 25.0201 38.8678	hor5 24.2285 33.8915 15.0037 17.8954 25.2527 37.6775	 	hor21 66.1543 55.0428 39.0670 37.7416 57.1504 65.5041	hor22 38.7290 64.8494 43.6679 51.4164 54.6704 71.3383	hor23 28.8213 54.8913 56.9772 33.2892 69.6278 63.1989	hor24 34.8032 42.2488 67.7810 27.3350 59.2099 64.8086
gretl h: NBR-/->r NBR-/->D NBR-/->GDP r-/->NBR r-/->P r-/->GDP P-/->NBR	hor1 38.5206 50.5512 27.8031 22.5395 32.2534 26.3351 17.0670	hor2 26.3851 45.2468 25.0114 20.8631 32.9285 28.6642 18.9469	hor3 24.3669 49.1373 25.5109 17.2362 32.0432 35.9767 16.6844	hor4 22.5679 33.8523 23.6785 17.4229 25.0201 38.8678 21.2361	hor5 24.2285 33.8915 15.0037 17.8954 25.2527 37.6775 28.7231	 	hor21 66.1543 55.0428 39.0670 37.7416 57.1504 65.5041 50.2713	hor22 38.7290 64.8494 43.6679 51.4164 54.6704 71.3383 45.8035	hor23 28.8213 54.8913 56.9772 33.2892 69.6278 63.1989 54.1886	hor24 34.8032 42.2488 67.7810 27.3350 59.2099 64.8086 56.7819
gretl h: NBR-/->r NBR-/->P NBR-/->GDP r-/->NBR r-/->GDP P-/->NBR P-/->r	hor1 38.5206 50.5512 27.8031 22.5395 32.2534 26.3351 17.0670 22.4393	hor2 26.3851 45.2468 25.0114 20.8631 32.9285 28.6642 18.9469 16.4460	hor3 24.3669 49.1373 25.5109 17.2362 32.0432 35.9767 16.6844 14.3076	hor4 22.5679 33.8523 23.6785 17.4229 25.0201 38.8678 21.2361 14.2927	hor5 24.2285 33.8915 15.0037 17.8954 25.2527 37.6775 28.7231 14.0141	 	hor21 66.1543 55.0428 39.0670 37.7416 57.1504 65.5041 50.2713 16.9652	hor22 38.7290 64.8494 43.6679 51.4164 54.6704 71.3383 45.8035 31.9379	hor23 28.8213 54.8913 56.9772 33.2892 69.6278 63.1989 54.1886 28.8408	hor24 34.8032 42.2488 67.7810 27.3350 59.2099 64.8086 56.7819 30.0024
gretl h: NBR-/->r NBR-/->D NBR-/->D r-/->NBR r-/->D P-/->NBR P-/->R P-/->C	hor1 38.5206 50.5512 27.8031 22.5395 32.2534 26.3351 17.0670 22.4393 20.6929	hor2 26.3851 45.2468 25.0114 20.8631 32.9285 28.6642 18.9469 16.4460 24.1138	hor3 24.3669 49.1373 25.5109 17.2362 32.0432 35.9767 16.6844 14.3076 27.4125	hor4 22.5679 33.8523 23.6785 17.4229 25.0201 38.8678 21.2361 14.2927 23.3593	hor5 24.2285 33.8915 15.0037 17.8954 25.2527 37.6775 28.7231 14.0141 22.9122	···· ··· ··· ··· ···	hor21 66.1543 55.0428 39.0670 37.7416 57.1504 65.5041 50.2713 16.9652 66.6108	hor22 38.7290 64.8494 43.6679 51.4164 54.6704 71.3383 45.8035 31.9379 59.2259	hor23 28.8213 54.8913 56.9772 33.2892 69.6278 63.1989 54.1886 28.8408 71.5332	hor24 34.8032 42.2488 67.7810 27.3350 59.2099 64.8086 56.7819 30.0024 67.2017
gretl h: NBR-/->r NBR-/->P NBR-/->GDP r-/->NBR r-/->P P-/->NBR P-/->CP GDP-/->NBR	hor1 38.5206 50.5512 27.8031 22.5395 32.2534 26.3351 17.0670 22.4393 20.6929 17.6338	hor2 26.3851 45.2468 25.0114 20.8631 32.9285 28.6642 18.9469 16.4460 24.1138 20.6570	hor3 24.3669 49.1373 25.5109 17.2362 32.0432 35.9767 16.6844 14.3076 27.4125 24.5341	hor4 22.5679 33.8523 23.6785 17.4229 25.0201 38.8678 21.2361 14.2927 23.3593 18.3230	hor5 24.2285 33.8915 15.0037 17.8954 25.2527 37.6775 28.7231 14.0141 22.9122 18.3132		hor21 66.1543 55.0428 39.0670 37.7416 57.1504 65.5041 50.2713 16.9652 66.6108 40.7055	hor22 38.7290 64.8494 43.6679 51.4164 54.6704 71.3383 45.8035 31.9379 59.2259 36.3952	hor23 28.8213 54.8913 56.9772 33.2892 69.6278 63.1989 54.1886 28.8408 71.5332 38.7162	hor24 34.8032 42.2488 67.7810 27.3350 59.2099 64.8086 56.7819 30.0024 67.2017 40.8469
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DPR package in GretI. Replication Results √



Causality measures. The Dufour & Taamouti (2010, DT) procedure

• Let $Y_t = (y_{1,t}, y_{2,t}, Y_{3,t})'$ the available detrended series where $y_{1,t}, y_{2t}$ are 1×1 and $Y_{3,t}$ is $m_3 \times 1$. Based on two VAR estimations, unrestricted and restricted with $Y_{0,t} = (y_{1,t}, Y_{3,t})'$ (drop the second variable from available information set)

$$Y_{t} = \sum_{k=1}^{p} \pi_{k} Y_{t-k} + u_{t} | Y_{0,t} = \sum_{k=1}^{p} \pi_{0,k} Y_{0,t-k} + e_{t}$$

1. Figure out *p* prior to building the causality measures. Keep \hat{u}_t , $\hat{\pi}_k$, k = 1, ..., p and the variance-covariance matrix $\hat{\Sigma}_{u|p}$ of the error term. Compute the $VMA(\infty)$ $\hat{\Psi}_j$, j = 0, ..., h - 1, $\hat{\Psi}_0 = I_K$, coefficient matrices by recursive substitution based on $\hat{\pi}_k$ and the unconstrained forecast error variance-covariance matrix $\hat{\Sigma}_p(h) = \sum_{j=0}^{h-1} \hat{\Psi}_j \hat{\Sigma}_{u|p} \hat{\Psi}'_j$ at horizon *h*.

Similarly, for the constrained model calculate $\tilde{\Sigma}_{0|p}(h) = \sum_{j=0}^{h-1} \tilde{\Psi}_j \tilde{\Sigma}_{e|p} \tilde{\Psi}_j'$

Causality measures. The Dufour & Taamouti (2010, DT) procedure

2. Causality measure (C.M) estimate is given by

$$\hat{C}_{L}(h) = \hat{C}_{L}\left(y_{2} \xrightarrow{h} y_{1}\right) = \ln\left(\frac{\left[\tilde{\Sigma}_{0|p}(h)\right]_{1,1}}{\left[\hat{\Sigma}_{p}(h)\right]_{1,1}}\right)$$

- 4. Residual-based bootstrap method proposed by DT to compute the confidence interval (lower bound, L.B & upper bound, U.B) of $\hat{C}_L(h)$ at a given horizon *h*. A mean bias correction is applied and a non-negativity truncation is imposed
- 3. Prior to bootstraping C.M, a separate bootstrap using \hat{u}_t , $\hat{\pi}_k$ is employed to correct the bias in the estimated VAR coefficients. In step 4, the bias corrected coefficients are employed

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DT package in Gretl

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DT package in GretI:GUI

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DT package in Gretl: output



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multi-horizon causality

DT package in Gretl. Replication results



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