Error Correction Models with Neglected Asymmetry

Artur Tarassow[1] Matthew Greenwood-Nimmo[2]

[1]Faculty of Social Sciences & Economics
University of Hamburg

[2]Faculty of Business & Economics
University of Melbourne

4th `gretl` Conference
June 12-13, 2015

Motivation

- Economic data are cointegrated as they respond to shocks together.
- Discussion of nonlinear cointegration in Park and Phillips (2001), threshold cointegration in Balke and Fomby (1997), and nonlinear adjustment mechanisms with asymmetric error correction in Enders and Granger (1998) and Enders and Siklos (2001).
- Type of asymmetry we are concerned with: Series respond only to a certain kind of shocks, e.g. due to downward rigidity or hysteresis effects (Schorderet, 2001, 2003; Granger and Yoon, 2002).
- Recently Shin, Yu and Greenwood-Nimmo (2014) have followed the framework of Schorderet (2001) by augmenting the standard ARDL(p,q) and to allow for long- and short-run asymmetries in the effects of certain shocks (**NARDL**).
  $\Rightarrow$ Linear adjustment but nonlinear cointegration relationship.

Motivation

- **Two contentions** haven been made in the literature on asymmetric cointegration:
  1. Long-run asymmetry can **confound efforts to test for a stable cointegrating relationship** if the test assumes long-run symmetry (linearity) (Schorderet, 2001).
  2. Mis-specifying an asymmetric long-run relationship as symmetric can profoundly **bias the dynamic parameter estimates** in the associated ECM (Shin et al., 2014).
- To date we are unaware of any published research that elaborates upon the nature of the biases and distortions imparted in this manner.
- We run a series of MC experiments.
- The importance is illustrated in the case of the nonlinear unemployment-output relationship in the U.S. economy.

Major result

Always start your application with the general asymmetric ECM!

The tests on

A) cointegration,
B) long-run symmetry,
C) short-run symmetry and
D) parameter estimates

are not robust against neglected asymmetry.

Outline

Following Schorderet (2001), consider the asymmetric bivariate cointegrating regression of y_t on x_t^+ and x_t^-:

$$y_t = \beta^+ x_t^+ + \beta^- x_t^- + \xi_t, \tag{1}$$

$$\Delta x_t = v_t, \tag{2}$$

where y_t and x_t are scalar I(1) variables and x_t is decomposed into its non-stationary components $x_t = x_0 + x_t^+ + x_t^-$ with:

$$x_t^+ = \sum_{j=1}^{t} \Delta x_j^+ = \sum_{j=1}^{t} \max\left(\Delta x_j, 0\right), \; x_t^- = \sum_{j=1}^{t} \Delta x_j^- = \sum_{j=1}^{t} \min\left(\Delta x_j, 0\right). \tag{3}$$

x_t^+ and x_t^+ are cumulative partial sum processes.

The asymmetric cointegration relationship in (1) can be written as a nonlinear ARDL(p,q) model and re-written as a conditional ECM (Pesaran and Shin, 1998; Shin et al., 2014):

$$\Delta y_t = \rho \xi_{t-1} + \sum_{j=1}^{p-1} \gamma_j \Delta y_{t-j} + \sum_{j=0}^{q-1} \left(\boldsymbol{\pi}_j^{+\prime} \Delta \boldsymbol{x}_{t-j}^{+} + \boldsymbol{\pi}_j^{-\prime} \Delta \boldsymbol{x}_{t-j}^{-} \right) + e_t \tag{4}$$

where $\xi_t = y_t - \boldsymbol{\beta}^{+\prime} \boldsymbol{x}_t^{+} - \boldsymbol{\beta}^{-\prime} \boldsymbol{x}_t^{-}$ is the nonlinear error correction term obtained from (1).

The **NARDL** model can be consistently estimated by **OLS** in a **single step**:

$$\Delta y_t = \rho y_{t-1} + \boldsymbol{\theta}^{+\prime} \boldsymbol{x}_{t-1}^{+} + \boldsymbol{\theta}^{-\prime} \boldsymbol{x}_{t-1}^{-} + \sum_{j=1}^{p-1} \gamma_j \Delta y_{t-j} + \sum_{j=0}^{q-1} \left(\boldsymbol{\pi}_j^{+\prime} \Delta \boldsymbol{x}_{t-j}^{+} + \boldsymbol{\pi}_j^{-\prime} \Delta \boldsymbol{x}_{t-j}^{-} \right) + e_t \tag{5}$$

Long-run multipliers: $\boldsymbol{\beta}^{+} \equiv -\boldsymbol{\theta}^{+}/\rho$ and $\boldsymbol{\beta}^{-} \equiv -\boldsymbol{\theta}^{-}/\rho$.

Major results I

1. Pesaran/Shin/Smith (PSS) **bounds test on cointegration is robust against mis-specification** of the **short-run dynamics** as long-as the long-run functional form is correctly specified.
2. Both the Engle-Granger (EG) as well as PSS cointegration tests **do not detect (partial) hidden cointegration**.
3. PSS test performs superior to the EG test if the underlying DGP is symmetric in finite samples. More complex pattern arise if the underlying DGP is fully asymmetric but restricted models (in the SR- or LR-parameters) are estimated, instead.
4. **Wald test on long-run symmetry** performs reasonable in small samples and is robust against mis-specification of the short-run dynamics.

Major results II

5. **Wald test on short-run symmetry** performs reasonable as long as the short-run dynamics are correctly specified – irrespective of how the long-run relationship is specified.
6. Application to Okun's Law illustrates that neglecting existent long-run asymmetry profoundly **distorts the long-run parameter estimates**.
7. Neglecting existent short-run asymmetries has substantial implications in terms of bias on the estimated dynamics.

Monte Carlo Simulations I

For investigation of the finite sample properties of the estimators and key test statistics under a variety of model mis-specifications, our **MC study** is based on the following **six specifications**:

*1. Asymmetric long-run relationship and asymmetric short-run dynamics (**ALR-ASR**)*

$$\Delta y_t = \rho \left(y_{t-1} - \boldsymbol{\beta}^{+\prime} \boldsymbol{x}_{t-1}^{+} + \boldsymbol{\beta}^{-\prime} \boldsymbol{x}_{t-1}^{-} \right) + \sum_{j=1}^{p-1} \gamma_j \Delta y_{t-j} + \sum_{j=0}^{q-1} \left(\boldsymbol{\pi}_j^{+\prime} \Delta \boldsymbol{x}_{t-j}^{+} + \boldsymbol{\pi}_j^{-\prime} \Delta \boldsymbol{x}_{t-j}^{-} \right) + e_t \quad (6)$$

*2. Asymmetric long-run relationship and symmetric short-run dynamics (**ALR-SSR**)*

$$\Delta y_t = \rho \left(y_{t-1} - \boldsymbol{\beta}^{+\prime} \boldsymbol{x}_{t-1}^{+} + \boldsymbol{\beta}^{-\prime} \boldsymbol{x}_{t-1}^{-} \right) + \sum_{j=1}^{p-1} \gamma_j \Delta y_{t-j} + \sum_{j=0}^{q-1} \boldsymbol{\pi}_j^{\prime} \Delta \boldsymbol{x}_{t-j} + e_t \quad (7)$$

*3. Symmetric long-run relationship and asymmetric short-run dynamics (**SLR-ASR**)*

$$\Delta y_t = \rho \left(y_{t-1} - \boldsymbol{\beta}^{\prime} \boldsymbol{x}_{t-1} \right) + \sum_{j=1}^{p-1} \gamma_j \Delta y_{t-j} + \sum_{j=0}^{q-1} \left(\boldsymbol{\pi}_j^{+\prime} \Delta \boldsymbol{x}_{t-j}^{+} + \boldsymbol{\pi}_j^{-\prime} \Delta \boldsymbol{x}_{t-j}^{-} \right) + e_t \quad (8)$$

Monte Carlo Simulations II

*4. Symmetric long-run relationship and symmetric short-run dynamics (**SLR-SSR**)*

$$\Delta y_t = \rho \left(y_{t-1} - \boldsymbol{\beta}' \boldsymbol{x}_{t-1} \right) + \sum_{j=1}^{p-1} \gamma_j \Delta y_{t-j} + \sum_{j=0}^{q-1} \boldsymbol{\pi}_j' \Delta \boldsymbol{x}_{t-j} + e_t \tag{9}$$

*5. No long-run relationship and asymmetric short-run dynamics (**XLR-ASR**)*

$$\Delta y_t = \sum_{j=1}^{p-1} \gamma_j \Delta y_{t-j} + \sum_{j=0}^{q-1} \left(\boldsymbol{\pi}_j^{+\prime} \Delta \boldsymbol{x}_{t-j}^{+} + \boldsymbol{\pi}_j^{-\prime} \Delta \boldsymbol{x}_{t-j}^{-} \right) + e_t \tag{10}$$

*6. No long-run relationship and symmetric short-run dynamics (**XLR-SSR**)*

$$\Delta y_t = \sum_{j=1}^{p-1} \gamma_j \Delta y_{t-j} + \sum_{j=0}^{q-1} \boldsymbol{\pi}_j' \Delta \boldsymbol{x}_{t-j} + e_t \tag{11}$$

For each of these six DGPs we estimate all six specifications under consideration. Thus, **in total 36 combinations are evaluated**.

Monte Carlo Simulations III

In all cases the marginal DGP for x_t follows a simple random walk process:

$$\Delta x_t = \varepsilon_t \ . \tag{12}$$

Serially uncorrelated realisations of e_t and ε_t from the following bivariate normal distribution are simulated:

$$\begin{pmatrix} e_t \\ \varepsilon_t \end{pmatrix} \sim N\left\{\mathbf{0}, \mathbf{\Omega} = \begin{pmatrix} 1 & \omega \\ \omega & 1 \end{pmatrix}\right\}. \tag{13}$$

Monte Carlo Simulations IV

We simulate all six DGPs for – Of course using `gretl`!:

1. $T \in \{75, 100, 200, 400, 1000\}$
2. The lag order $p = q = 1$ (in levels) is assumed to be known.
3. The constant, α, is assumed to be zero.
4. Simulations are conducted for different values of $\omega \in \{-0.5, 0, 0.5\}$ and $\rho \in \{-0.1, -0.2, -0.4, -0.6\}$.
5. Throughout it is assumed that $\beta^{+} = 0.5$ but the long-run coefficient associated with negative changes in x, $\beta^{-} = \beta^{+} + \delta_{\beta}$, differs as $\delta_{\beta} \in \{0.1, 0.3, 0.5\}$.
6. We also vary the parameter values associated with the contemporaneous short-term dynamics, for which it is assumed that $\theta^{+} = 0.5$ and $\theta^{-} = \theta^{+} + \delta_{\theta}$, and $\delta_{\theta} \in \{0.1, 0.3, 0.5\}$.

All experiments are run over 3,000 replications but any replications for which the stability condition $\rho <= 10^{-4}$ is not met, is discarded.

Monte Carlo Simulations V

We investigate the **frequency to reject the null hypothesis** of:[1]

- Long-run symmetry:
$$H_{LR}^{S} : \beta^{+} = \beta^{-} \tag{14}$$
- Symmetric short-run dynamics:
$$H_{SR}^{S} : \pi^{+} = \pi^{-} \tag{15}$$
- PSS bounds test of the null of no asymmetric cointegration:
$$H_{PSS}^{AS} : \rho = \beta^{+} = \beta^{-} = 0 \tag{16}$$
- PSS bounds test of the null of no symmetric cointegration:
$$H_{PSS}^{S} : \rho = \beta = 0 \tag{17}$$
- Engle-Granger residual-based cointegration approach using the ADF unit root test.

[1] All following tests are conducted at the nominal 0.05 significance level.

Major results

- If the model is correctly specified, the rejection frequency (RF) increases strongly in sample size T.
- RF are positively correlated with the speed of long-run adjustment $|\rho|$ but only marginally on ω.
- For samples below $T < 200$ and slow error-correction adjustments the RF of the PSS F-test is rather low.
- Short-run mis-specification does not affect the overall results.

$$H_{PSS}^{AS} : \rho = \beta^{+} = \beta^{-} = 0$$

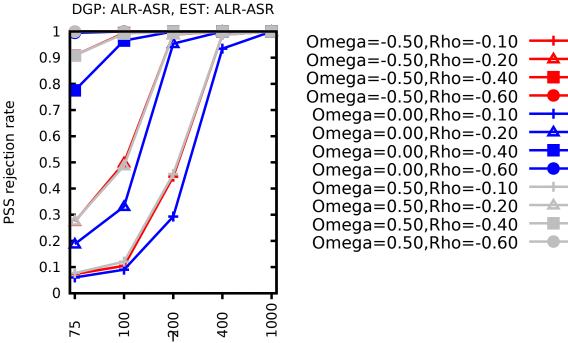


Figure : DGP: $ALR - ASR$, EST: $ALR - ASR$

PSS bounds test II

Major results

- PSS test is unable to detect the nonlinear cointegrating relationship in case the estimated model is restricted to be long-run symmetric.
- The rejection frequencies wrongly decrease in T. $\Rightarrow$ partial **Hidden Cointegration** phenomena
- Result holds irrespective of short-run mis-specifications.

$$H^S_{PSS} : \rho = \beta = 0$$

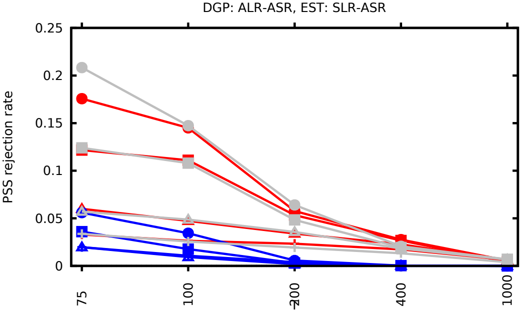


Figure : DGP: $ALR - ASR$, EST: $SLR - ASR$

Major results

- Strong reasons to start with the most general specification if the underlying DGP is unknown.
- In case the DGP is fully symmetric but a (long-run) asymmetric model is estimated, the performance of the PSS test is still reasonable.
- Again frequencies increase in T as well as $|\rho|$.

$$H_{PSS}^{AS} : \rho = \beta^{+} = \beta^{-} = 0$$

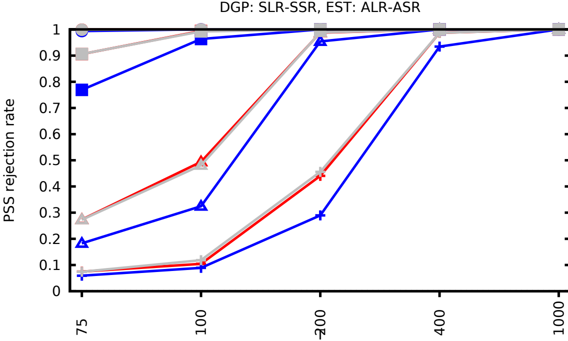


Figure : DGP: $SLR - SSR$, EST: $ALR - ASR$

Major results

- If the underlying DGP allows for long-run asymmetry, the test performance is robust against the misspecification of short-run dynamics.
- Again the RF increases in T as well as $|\rho|$, but slightly decreases in $|\omega|$.

$$H_{LR}^{S} : \beta^{+} = \beta^{-}$$

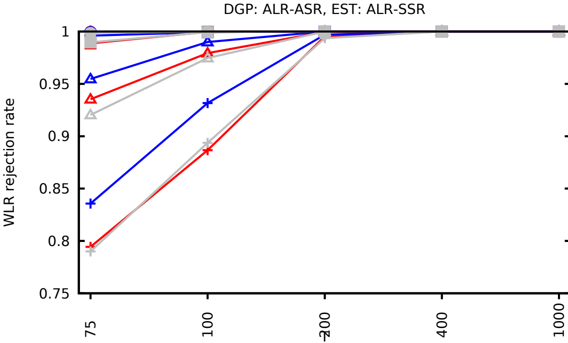


Figure : DGP: $ALR-ASR$, EST: $ALR-SSR$

Major results

- Again the RF increases in T as well as $|\rho|$ if the DGP is symmetric in the long-run relationship.
- For instance, if DGP=$SLR - ASR$ but a fully-asymmetric model is estimated, the RF is about 5% in limit and slightly oversized in small samples.
- This holds irrespective of the functional form of the dynamics estimated.
- Overall, the Wald test performs well.

$$H_{LR}^{S} : \beta^{+} = \beta^{-}$$

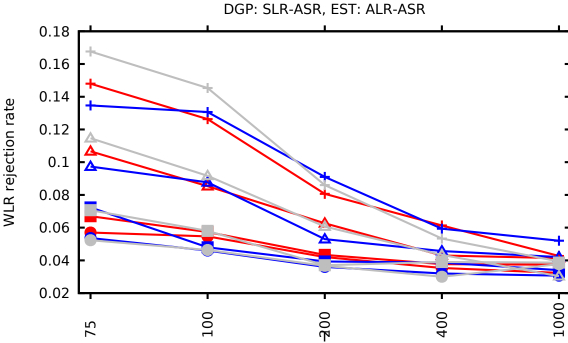


Figure : DGP: $SLR - ASR$, EST: $ALR - ASR$

Major results

- In general, the test performs reasonable for DGP with asymmetric short-run dynamics as long as the dynamics of the estimated model are correctly specified.
- This holds irrespective of long-run misspecification.
- The RF increases strongly in T, and decreases slightly in $|\rho|$ and $|\omega|$.

$$H_{SR}^{S} : \pi^{+} = \pi^{-}$$

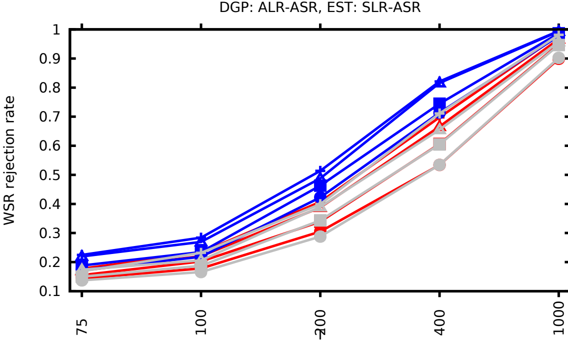


Figure : DGP: $ALR-ASR$, EST: $SLR-ASR$

Major results

- Size of the test is close to the nominal 5% level as long as the short-run dynamics are correctly specified; even for small samples.
- Holds irrespective of the mis-specification of the long-run relationship.

$$H_{SR}^{S}: \pi^{+} = \pi^{-}$$

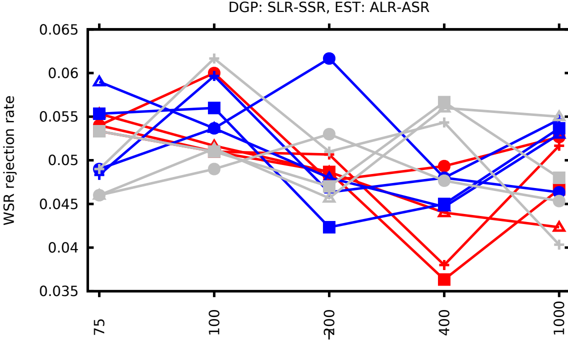
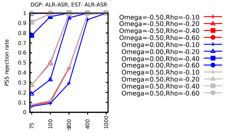
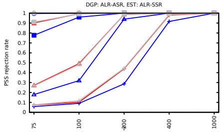


Figure : DGP: $SLR - SSR$, EST: $ALR - ASR$

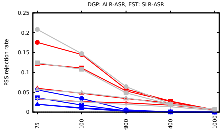
PSS bounds test



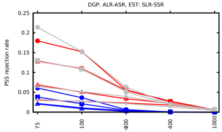
(a)



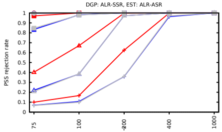
(b)



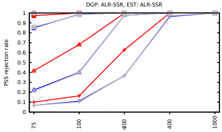
(c)



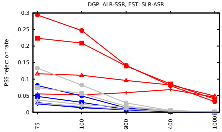
(d)

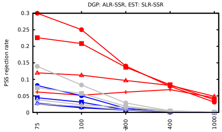


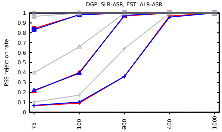
(e)



(f)







An Application to Okun's Law I

- Shin et al. (2014) find **evidence of pronounced asymmetry** in both the long- and the short-run dynamics with a pattern that is consistent with **hysteretic adjustment** in the US labour market.
- We conduct a kind of **counter-factual analysis** by estimating Okun's Law relationship under four different specifications:
  1) $ALR - ASR$, 2) $ALR - SSR$, 3) $SLR - ASR$ and 4) $SLR - SSR$
- Monte Carlo study to evaluate which effect estimating the wrong model has for coefficient estimates, inference and obtained dynamics by means of **dynamic multiplier analysis**.

An Application to Okun's Law II

The MC study is based on the following algorithm:

1. Estimate Okun's Law based on the first case ($ALR - ASR$) and save the residuals and coefficient estimates.
2. Generate a semi-parametric bootstrap sample by taking the following as given: values of the output gap; sufficient initial values of unemployment to account for the lag structure in the model; the coefficient estimates.
3. Estimate all four cases on the bootstrap sample of data subject to G-2-S lag selection as normal. Save the parameter estimates and key test statistics. Also compute the dynamic multipliers for each estimated model.
4. Repeat steps 2 to 3 3,000 times under consideration of the stability requirement that $\rho <= -0.001$.
5. Generate empirical confidence intervals for the following key parameters and test statistics: error correction coefficient, long-run coefficients, dynamic multipliers at selected horizons, R-square, *PSS*, *WLR*, *WSR* and *EG* tests.
6. Repeat using the other three cases as DGPs.

An Application to Okun's Law III – Table

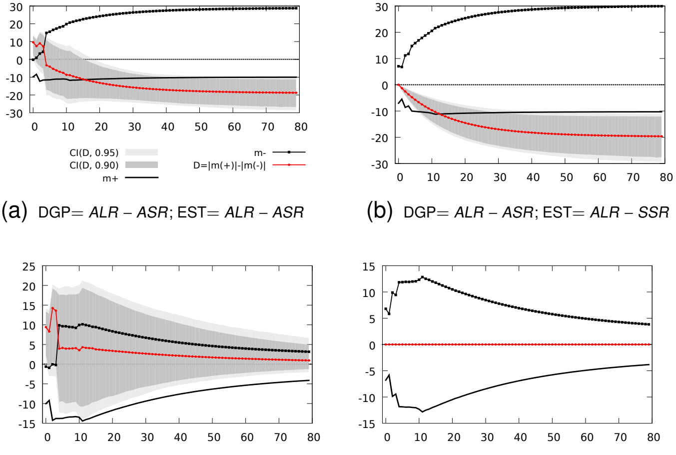
	EST = 1	EST = 2	EST = 3	EST = 4
ρ	−0.065	−0.066	−0.029	−0.028
	(−0.11/−0.04)	(−0.11/−0.04)	(−0.05/−0.01)	(−0.05/−0.01)
β			−1.982	−1.966
			(−3.78/2.07)	(−3.82/2.29)
β^+	−9.984	−10.249		
	(−13.51/−6.51)	(−13.74/−6.91)		
β^-	−28.784	−29.852		
	(−42.84/−16.60)	(−43.77/−18.14)		
$m_1^+ - m_1^-$	−9.909	0.000	−10.101	
	(−20.74/−4.02)	(0.00/0.00)	(−23.24/−5.34)	
$m_3^+ - m_3^-$	−9.329	2.477	−14.792	
	(−24.45/6.96)	(1.20/4.22)	(−29.33/2.82)	
$m_6^+ - m_6^-$	4.546	5.606	−3.389	
	(−11.62/20.62)	(2.77/9.19)	(−21.50/12.32)	
$m_{12}^+ - m_{12}^-$	8.886	10.245	−3.901	
	(−7.87/24.99)	(5.27/15.70)	(−22.88/13.89)	
R^2	0.330	0.294	0.297	0.261
	(0.25/0.42)	(0.22/0.38)	(0.22/0.38)	(0.19/0.34)
F_{FF}	0.05	0.06	0.05	0.11
$F_{PSS(k=1/2)}$	0.87/0.95	0.87/0.95	0.38	0.32
$EG_{(k=1/2)}$	0.00/0.00	0.00/0.00	0.00	0.00
W_{LR}	1.00	1.00		
W_{SR}	0.36		0.33	

Table : Estimation of the Unemployment-Output Relationship assuming that the actual DGP follows $ALR - ASR$

Cumulative dynamic multiplier effect of x_t^+ on y_t:

$$m_h^+ = \sum_{j=0}^{h} \frac{\partial y_{t+j}}{\partial x_t^+}$$

y – unemployment rate
x – log of industrial production



(a) DGP= $ALR - ASR$; EST= $ALR - ASR$

(b) DGP= $ALR - ASR$; EST= $ALR - SSR$

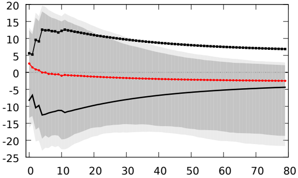
(c) DGP= $ALR - ASR$; EST= $SLR - ASR$

(d) DGP= $ALR - ASR$; EST= $SLR - SSR$

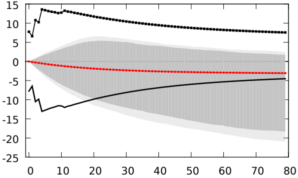
Figure : US Unemployment-Output Dynamic Multipliers

	EST = 1	EST = 2	EST = 3	EST = 4
ρ	−0.033	−0.033	−0.030	−0.030
	(−0.07/−0.01)	(−0.07/−0.01)	(−0.06/−0.01)	(−0.06/−0.01)
β			−2.017	−2.077
			(−5.15/5.33)	(−5.17/4.75)
β^+	−3.391	−3.596		
	(−10.25/8.82)	(−10.42/7.67)		
β^-	−6.775	−7.464		
	(−32.20/28.47)	(−32.89/25.65)		
$m_1^+ - m_1^-$	−6.762	0.000	−7.038	
	(−13.21/16.33)	(0.00/0.00)	(−13.22/16.20)	
$m_3^+ - m_3^-$	−0.203	0.247	−0.873	
	(−18.15/21.20)	(−0.93/1.74)	(−18.51/20.16)	
$m_6^+ - m_6^-$	−0.122	0.578	−1.528	
	(−20.67/22.40)	(−2.20/4.05)	(−21.29/20.95)	
$m_{12}^+ - m_{12}^-$	0.969	1.154	−1.135	
	(−22.24/22.79)	(−4.38/7.61)	(−22.02/21.42)	
R^2	0.294	0.283	0.289	0.279
	(0.20/0.39)	(0.20/0.37)	(0.20/0.38)	(0.19/0.37)
$F_{SC(1)}$	0.00	0.00	0.00	0.00
$F_{SC(4)}$	0.00	0.00	0.00	0.00
χ_H^2	0.07	0.08	0.07	0.07
χ_N^2	0.25	0.28	0.26	0.29
F_{FF}	0.09	0.08	0.08	0.07
$F_{PSS(k=1/2)}$	0.37/0.55	0.35/0.53	0.67	0.67
$EG_{(k=1/2)}$	0.00/0.00	0.00/0.00	0.00	0.00
W_{LR}	0.95	0.95		
W_{SR}	0.35		0.34	

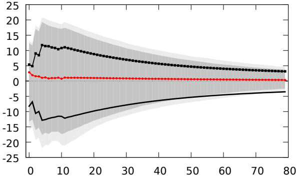
Table : Estimation of the Unemployment-Output Relationship assuming that the actual DGP follows $SLR - SSR$



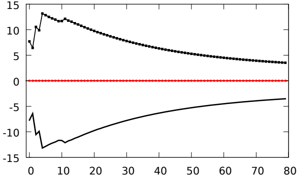
(a) DGP= *SLR* – *SSR*; EST= *ALR* – *ASR*



(b) DGP= *SLR* – *SSR*; EST= *ALR* – *SSR*



(c) DGP= *SLR* – *SSR*; EST= *SLR* – *ASR*



(d) DGP= *SLR* – *SSR*; EST= *SLR* – *SSR*

Figure : US Unemployment-Output Dynamic Multipliers

- Practitioner should always start with the general fully asymmetric model in case the functional form is unknown.
- *PSS* bounds test is robust against mis-specification of the short-run dynamics.
- Both the *EG* and *PSS* test do not detect partial *hidden cointegration*.
- RF of the *Wald test on long-run symmetry* (WLR) shows reasonable power and size properties.
- *WLR* is robust against mis-specificaiton of short-run dynamics.
- *Wald test on short-run symmetry* is robust against the mis-specification of long-run relationship.
- Application illustrates substantial long- and short-run coefficient distortions if the underlying DGP is fully asymmetric but restricted cases are estimated. **But not** *vice versa*.

References I

Balke, N. and T. B. Fomby (1997). Threshold cointegration. *International Economic Review 38*(3), 627–645.

Enders, W. and C. W. J. Granger (1998, July). Unit-Root Tests and Asymmetric Adjustment with an Example Using the Term Structure of Interest Rates. *Journal of Business & Economic Statistics 16*(3), 304–11.

Enders, W. and P. L. Siklos (2001, April). Cointegration and Threshold Adjustment. *Journal of Business & Economic Statistics 19*(2), 166–76.

Granger, C. W. and G. Yoon (2002). Hidden cointegration. unpublished manuscript, UC San Diego: Department of Economics, UCSD.

Park, J. Y. and P. C. B. Phillips (2001, January). Nonlinear regressions with integrated time series. *Econometrica 69*(1), 117–61.

Pesaran, M. H. and Y. Shin (1998). An autoregressive distributed lag modelling approach to cointegration analysis. In S. Strom (Ed.), *Econometrics and Economic Theory: The Ragnar Frisch Centennial Symposium. Econometric Society Monographs.*, pp. 371–413. Cambrdige: Cambridge University Press.

Schorderet, Y. (2001). Revisiting okun's law: An hysteretic perspective. Mimeo, University of California San Diego.

References II

Schorderet, Y. (2003). Asymmetric cointegration. Technical report, University of Geneva. unpublished manuscript, University of Geneva.

Shin, Y., B. Yu, and M. Greenwood-Nimmo (2014). Modelling asymmetric cointegration and dynamic multipliers in a nonlinear ardl framework. In W. Horrace and R. Sickles (Eds.), *Festschrift in Honor of Peter Schmidt: Econometric Methods and Applications*. New York: Springer Science and Business Media.