BMA related packages in gretl

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We consider here three BMA type packages in `gretl`:

1. **BMA** (ver. 2.01)[1].
2. **BACE** (ver. 1.1) and unreleased ver. 2.0 (with MPI)[2].
3. **BMA_ADL** (unreleased) with MPI[3].

[1] Błażejowski, M., & Kwiatkowski, J. (2015). Bayesian Model Averaging and Jointness Measures for gretl. *Journal of Statistical Software*, *68*(5), 1–24.

[2] Błażejowski, M., & Kwiatkowski, J. (2018). *Bayesian Averaging of Classical Estimates (BACE) for gretl* (gretl working papers No. 6). Universita' Politecnica delle Marche (I), Dipartimento di Scienze Economiche e Sociali.

[3] Błażejowski, M., & Kwiatkowski, J. (2020). *Bayesian Model Averaging for Autoregressive Distributed Lag (BMA_ADL) in gretl* (MPRA Paper No. 98387). University Library of Munich, Germany.

gretl: function packages on server

All packages | filter

Package	Version	Date	Author	Summary	Local status
addlist	2.0	2020-08-15	Allin Cottrell	Sequential addition of variables to a model	Not up to date
ADFGLSTest	0.1	2020-11-15	Oleh Komashko	ADF-GLS test with interpolated p-values	Not up to date
ADMBP	1.0	2021-04-08	Artur Tarassow	ARDL Dynamic Multiplier Bootstrap Package	Not up to date
almonreg	0.4	2015-02-09	Allin Cottrell	PDL (Almon lag) model	Up to date
arima_sim	0.23	2018-11-12	Oleh Komashko	simulation of arima(p, d, q; P, D, Q) traject...	Not installed
armax	0.101	2017-09-22	Yi-Nung Yang	Automatically determine and show the best ARM...	Not installed
ARMA_IRF	1.0	2020-04-27	Riccardo "Jack" Lucchetti	IRF for univariate ARMA models	Not installed
ARprobit	0.7	2021-02-21	Sven Schreiber	Dynamic Probit (univariate/AR, Kauppi&Saikkonen)	Not installed
assertion	1.0	2020-08-22	Artur Tarassow	Assert functions for verifying expectations a...	Not installed
auto_arima	0.7	2020-09-03	Artur Tarassow	Return best ARIMA model according to informat...	Not installed
a_eff	0.9	2020-11-15	Oleh Komashko	advanced treatment of marginal effects and el...	Not installed
BACE	1.1	2018-08-31	Marcin Błażejowski, Jac...	This package provides BACE (Bayesian Averagin...	Up to date
bandplot	0.3	2015-07-11	Allin Cottrell	Confidence band plot	Not installed
bartlett	0.3	2017-07-02	Marcos Larios Santa Rosa	Bartlett's test for Homogeneity of Variance	Not installed
BMA	2.01	2017-06-29	Marcin Błażejowski, Jac...	Bayesian Model Averaging for the linear regre...	Up to date
BMST	2.1	2021-02-03	Artur Tarassow	Binary (logit + probit) model specification t...	Not up to date
BNdecomp	2.0	2015-03-03	Riccardo "Jack" Lucchetti	Beveridge-Nelson Decomposition	Not installed
BoxCoxFuncForm	0.3	2018-10-16	Sven Schreiber and Jürg...	Best-fitting functional form	Up to date
Box_Cox_Value	1.01	2014-05-13	Pedro Isaac Chávez López	Obtain the lambda value of a Box-Cox Transfor...	Not installed
BreitungCandelonTest	2.6	2021-01-19	Sven Schreiber	Breitung-Candelon test of frequency-wise Gran...	Not installed
Brown	1.5	2015-02-18	Ignacio Diaz-Emparanza	Brown linear and quadratic trend models	Not installed
buys_ballot	2.0	2018-02-15	Ignacio Diaz-Emparanza,...	Plots for seasonal time series	Not installed
calendar_utils	0.2	2020-10-16	Artur Tarassow	Collection of useful date time related tools	Not installed

Network status: OK

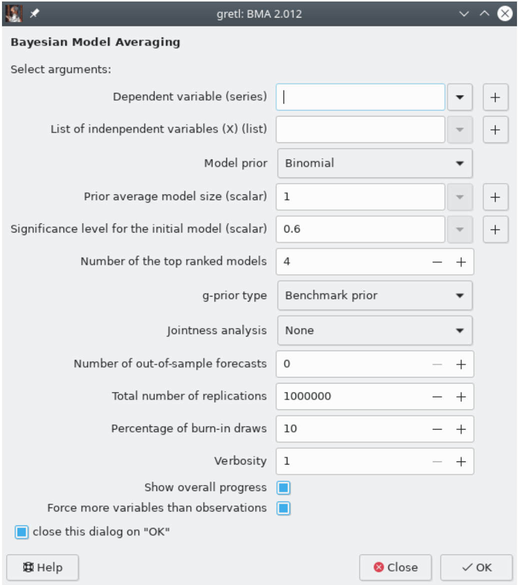
In our BMA package we use exactly the same prior structure as it was in:

- Fernández, C., Ley, E., & Steel, M. F. J. (2001a). Benchmark Priors for Bayesian Model Averaging. *Journal of Econometrics*, *100*, 381–427
- Fernández, C., Ley, E., & Steel, M. F. J. (2001b). Model uncertainty in cross-country growth regressions. *Journal of Applied Econometrics*, *16*(5), 563–576

+ **MC³** sampling algorithm[4].

[4] Madigan, D., York, J., & Allard, D. (1995). Bayesian Graphical Models for Discrete Data. *International Statistical Review*, *63*(2), 215–232.

GUI


gretl: BMA 2.012
Bayesian Model Averaging
Select arguments:
Dependent variable (series)
List of indenpendent variables (X) (list)
Model prior
Binomial
Prior average model size (scalar)
1
Significance level for the initial model (scalar)
0.6
Number of the top ranked models
4
g-prior type
Benchmark prior
Jointness analysis
None
Number of out-of-sample forecasts
0
Total number of replications
1000000
Percentage of burn-in draws
10
Verbosity
1
Show overall progress
Force more variables than observations
close this dialog on "OK"
Help
Close
OK

We can specify the following entries in the GUI **BMA** window

1. Loading variables from the database, which must have been opened previously.
    - **Dependent variable**
    - **List of independent variables**
2. Prior structure
    - **g-prior type** – One can choose between four Zellner's g-priors for the regression coefficients[5]. Choices include: benchmark prior, unit information prior, risk inflation criterion, Hannan and Quinn prior, root of g-UIP.
    - **Model prior** – Indicates the choice of model prior. One can employ the binomial model prior or the binomial-beta model prior. Note that the uniform model prior is a special case of the binomial model prior. Therefore, in fact, our package allows for three types of priors.

[5] Zellner, A. (1986). On Assessing Prior Distributions and Bayesian Regression Analysis with g-Prior Distributions. In P. Goel & A. Zellner (Eds.), *Bayesian inference and decision techniques: Essays in honor of bruno de finetti*. Elsevier.

2. Prior structure (continued)
   - **Prior average model size** – Specifies the prior expected model size $\mathsf{E}(\Xi)$. The expected model size may range from 0 to K (default: 1). Small value means that we penalize large models and we assign high probability to small ones following Occam's razor philosophy.
     Note that for the binomial model prior and with $\mathsf{E}(\Xi) = 0.5K$ we have the uniform prior on the model space.
3. **Significance level for the initial model** – Defines the significance level which is used to build the initial model. An explanatory variable enters the initial model if its p value is less than the significance level. If the significance level equals 1, the initial model will be randomly chosen (with equal probability) from all available models.

4. **Number of top ranked models** – Specifies the number of best models for which detailed information is stored.
5. **Jointness analysis** – If `"None"` (the default), the jointness analysis is omitted. Alternatively, one can choose the jointness measures of Ley-Steel[6] or Doppelhofer-Weeks[7].
6. **Number of out-of-sample forecasts** – Defines the total number of out-of-sample forecasts of the dependent variable.

[6] Ley, E., & Steel, M. F. J. (2007). Jointness in Bayesian Variable Selection with Applications to Growth Regression. *Journal of Macroeconomics*, *29*(3), 476–493.

[7] Doppelhofer, G., & Weeks, M. (2009). Jointness of Growth Determinants. *Journal of Applied Econometrics*, *24*(2), 209–244.

1. **Total number of replications** – Defines the total number of iteration draws to be sampled.
2. **Percentage of burn-in draws** – Specifies the number of burn-in replications, calculated as the percentage of the total number of iteration draws.
3. **Verbosity** – An integer value of 1 or 2; the default is 1, which allows to see the basic Bayesian model averaging results. If Verbosity equals 2, a more detailed description of the analysis is provided (initial model, speed of convergence, estimation results for top ranked models).

The **BMA_ADL** package implements Bayesian Model Averaging for Autoregressive Distributed Lag models.

We use $\mathbf{MC^3}$ sampling algorithm and similar prior structure to **BMA**.

The main difference is that we impose stationarity restrictions for autoregressive parameters.

The posterior for regression coefficients

Assuming the our prior structure we obtain the following joint posterior density:

$$p\left(\beta_r, h \mid y, M_r\right) = c_r \cdot f_{NG}\left(\beta_r, h \mid \overline{\beta}_r, \overline{V}_r, \overline{s}_r^{-2}, \overline{v}_r\right) I\left(\beta_r^y \in \Gamma\right),$$

where c_r is normalizing constant and f_{NG} is normal-gamma density[8].

In our case constant c_r plays important role to obtain the **Bayes factor** between competitive models and can be calculated as an inverse of the acceptance rate i.e. inverse of the fraction of random numbers accepted in Monte Carlo simulation.

[8] Koop, G., Poirier, D. J., & Tobias, J. L. (2007). *Bayesian Econometric Methods*. Cambridge University Press.

Single iteration of $\mathbf{MC^3}$ algorithm:

Step 1 Draw the candidate model M'.

Step 2 Take a random draw $h^{(m)} \mid M'$ from posterior $G\left(\overline{s}^{-2}, \overline{v}\right)$. G denotes *Gamma* distribution with mean $\overline{s}^{-2}$ where degrees of freedom $\overline{v}$.

Step 3 Take a random draw $\beta^{(m)} \mid h^{(m)}, M'$ from $N\left(\overline{\beta}, \overline{V}\right)$ where N denotes *Normal* distribution with mean $\overline{\beta}$ and covariance matrix $\overline{V}$.

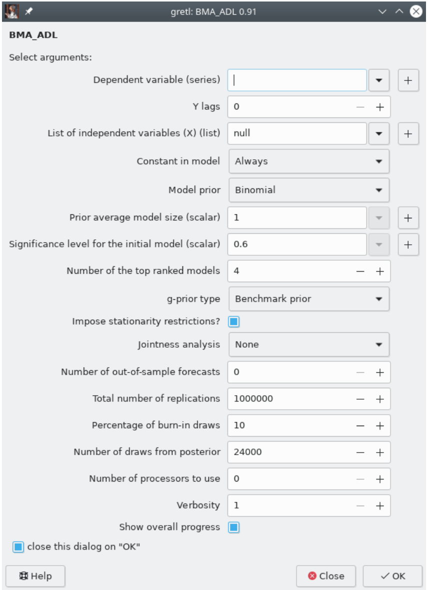
Step 4 Repeat Steps 1-2 m times and reject draws from the nonstationary region.

Step 5 Take the accepted draws to give the estimate of $\mathsf{E}(\beta \mid y)$ and $\mathsf{VAR}(\beta \mid y)$.

Step 6 Calculate normalizing constant c' for M'.

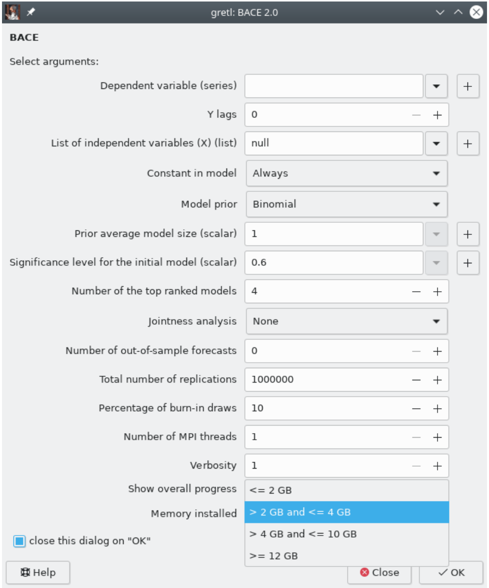
Step 7 Calculate $\mathsf{P}\left(M' \mid y\right)$.

Step 8 Accept the model M' with probability $\alpha\left(M^{(s-1)}, M'\right)$ using MC^3 algorithm and go to Step 1.


gretl: BMA_ADL 0.91
BMA_ADL
Select arguments:
Dependent variable (series)
Y lags
0
List of independent variables (X) (list)
null
Constant in model
Always
Model prior
Binomial
Prior average model size (scalar)
1
Significance level for the initial model (scalar)
0.6
Number of the top ranked models
4
g-prior type
Benchmark prior
Impose stationarity restrictions?
Jointness analysis
None
Number of out-of-sample forecasts
0
Total number of replications
1000000
Percentage of burn-in draws
10
Number of draws from posterior
24000
Number of processors to use
0
Verbosity
1
Show overall progress
close this dialog on "OK"
Help
Close
OK

We can specify the following additional (comparing to **BMA**) entries in the GUI window:

1. **Y lags** as integer with zero (only for time-series).
2. **Constant in model** as integer [required]:
   1. Never – all models without constant,
   2. Always – all models with constant,
   3. Can be dropped – constant may be removed from or added to any model.
3. **Impose stationarity restrictions?** [Yes/No]
4. **Number of draws from posterior** – Defines the number of draws samples of slope parameters from posterior distribution.
5. **Number of processors to use** – Defines the number of CPUs (phisical or logical) to use when drawing from posterior. Default value 0 means that package use maximum available CPUs.


gretl: BACE 2.0
BACE
Select arguments:
Dependent variable (series)
Y lags
0
List of independent variables (X) (list)
null
Constant in model
Always
Model prior
Binomial
Prior average model size (scalar)
1
Significance level for the initial model (scalar)
0.6
Number of the top ranked models
4
Jointness analysis
None
Number of out-of-sample forecasts
0
Total number of replications
1000000
Percentage of burn-in draws
10
Number of MPI threads
1
Verbosity
1
Show overall progress
Memory installed
<= 2 GB
> 2 GB and <= 4 GB
> 4 GB and <= 10 GB
>= 12 GB
close this dialog on "OK"
Help
Close
OK

BACE – Bayesian Averaging of Classical Estimates[9] approach **is not purely** Bayesian because it is based on OLS estimates.

[9] Sala-i-Martin, X., Doppelhofer, G., & Miller, R. I. (2004). Determinants of Long-Term Growth: A Bayesian Averaging of Classical Estimates (BACE) Approach. *American Economic Review*, *94*(4), 813–835.

The posterior probability of model M_r is **approximated** by Schwarz criterion:

$$P(M_r \mid y) \approx \frac{P(M_r) N^{-k_r/2} SSE_l^{-N/2}}{\sum_{l=1}^{2^K} P(M_l) N^{-k_l/2} SSE_l^{-N/2}},$$

where SSE_r and SSE_l are the OLS sum of squared errors.

Let us consider the data used in Fernández, C., Ley, E., & Steel, M. F. J. (2001b). Model uncertainty in cross-country growth regressions. *Journal of Applied Econometrics*, *16*(5), 563–576 (denoted FLS hereafter).

This data comprises the information about 72 countries and 41 potential growth determinants for the period 1960 to 1992 (demeaned).

Model space: $2^{41} = 2,199,023,255,552$ linear combinations.

Calling analysis via script

```
# BMA
BMA_res = BMA_GUI(Growth, Determinants, model_prior, prior_avg_model_size, \
        alpha, top_ranked_model, g_prior, jointness, forecasts, n_MC3, burnin, \
        verbosity, show_progress, 0)
# BACE
BACE_res = BACE_GUI(Growth, 0, Determinants, const_type, model_prior, \
        prior_avg_model_size, alpha, top_ranked_model, jointness, forecasts, \
        n_MC3, burnin, n_MPI, verbosity, show_progress, installed_RAM)
```

Variable	BMA[10] PIP	BMA[10] Avg. Mean	BMA[10] Avg. SD	BACE PIP	BACE Avg. Mean	BACE Avg. SD
Log GDP in 1960	0.99	-1.4239	0.2779	1.00	-1.5159	0.2412
Fraction Confucian	0.99	0.4929	0.1279	0.99	0.6108	0.1047
Life expectancy	0.93	0.9604	0.3939	0.99	1.0269	0.2494
Equipment investment	0.93	0.5552	0.2348	0.99	0.4750	0.1555
Sub-Saharan dummy	0.73	-0.4679	0.3483	0.99	-0.8718	0.2156
...			...			
Fraction Jewish	0.03	-0.0025	0.0275	0.12	-0.0020	0.0331
Area (scale effect)	0.03	-0.0009	0.0211	0.13	-0.0042	0.0454
Revolutions and coups	0.03	-0.0001	0.0224	0.12	0.0005	0.0359

[10] Błażejowski, M., & Kwiatkowski, J. (2015). Bayesian Model Averaging and Jointness Measures for gretl. *Journal of Statistical Software*, *68*(5), 1–24.

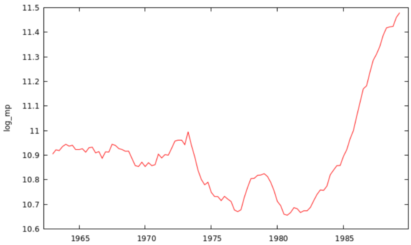
Comparison of **BMA_ADL** with the code by G. Koop, E. Ley, J. Osiewalski and M. Steel, ver. 24/April/95 compiled by1 gfortran-7.3.0 with options: ”-g -O3 -finit-local-zero”[11].

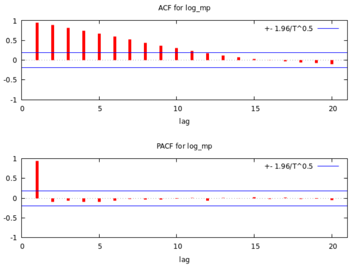
To do this, we use data concerning demand for narrow money (M1) in UK[12].

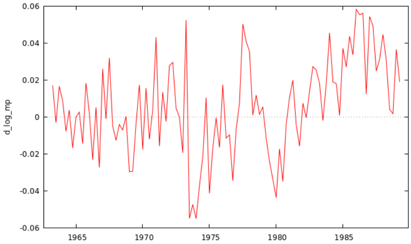
[11] Koop, G., Ley, E., Osiewalski, J., & Steel, M. F. (1997). Bayesian analysis of long memory and persistence using ARFIMA models. *Journal of Econometrics*, *76*(1-2), 149–169.

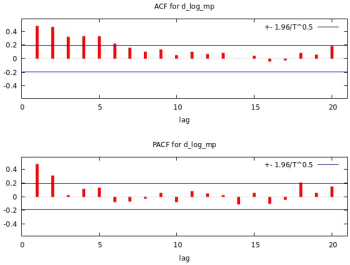
[12] Hendry, D. F., & Ericsson, N. R. (1991). 'Modelling the Demand for Narrow Money in the United Kingdom and the United States. *European Economic Review*, *35*(4), 833–881.

Time series plot: $\log(mp)$ and $\Delta \log(mp)$









We assume the following AR(p) model:

$$\Delta \log(mp)_t = \sum_{s=1}^{p} \beta_s \Delta \log(mp)_{t-s} + \varepsilon_t.$$

	AR(p)	$\mathbf{p(M_r)}$	$\hat\beta_1$		$\hat\beta_2$		$\hat\beta_3$	
			Mean	**SD**	**Mean**	**SD**	**Mean**	**SD**
arfima.f	AR(1)	0.022	0.4941	0.0886				
	AR(2)	0.720	0.341	0.0943	0.3236	0.0950		
	AR(3)	0.258	0.3214	0.0982	0.3445	0.1032	0.0273	0.1022
gretl	AR(1)	0.016	0.4872	0.0835				
	AR(2)	0.752	0.3389	0.0904	0.3115	0.0909		
	AR(3)	0.232	0.3335	0.0955	0.3053	0.0966	0.0163	0.0966

Longterm UK inflation

In the third empirical example, we used the long-term UK inflation model developed for 1875–1991 ($T = 117$)[13].

Variable	Definition	Variable	Definition
Y_t	real GDP, £ million, 1985 prices	$P_{e,t}$	world prices (1985 = 1)
P_t	implicit deflator of GDP (1985 = 1)	E_t	annual-average effective exchange rate
M_t	nominal broad money, million £	$P_{nni,t}$	deflator of net national income (1985 = 1)
$R_{s,t}$	three-month treasury bill rate	$P_{cpi,t}$	consumer price index (1985 = 1)
$R_{l,t}$	long-term bond interest rate	$P_{o,t}$	commodity price index, $
$R_{n,t}$	opportunity cost of money measure	m_t^d	money excess demand
N_t	nominal National Debt, £ million	y_t^d	GDP excess demand
U_t	unemployment	S_t	short–long spread
$Wpop_t$	working population	n_t^d	excess demand for debt
$U_{r,t}$	unemployment rate, fraction	$e_{r,t}$	real exchange rate
L_t	employment	π_t^*	profit markup
K_t	gross capital stock	U_t^d	excess demand for labor
W_t	wages	$p_{o,t}$	commodity prices in Sterling
H_t	normal hours (from 1920)	C_t	nominal unit labor costs

[13] Hendry, D. F. (2001). Modelling UK Inflation, 1875–1991. *Journal of Applied Econometrics*, *16*, 255–275.

The final specification after a number of variable transformations and model pre-reduction was as follows:

$$\Delta p_t = f(\Delta p_{t-1}, y^d_{t-1}, m^d_{t-1}, n^d_{t-1}, U^d_{t-1}, S_{t-1}, R_{l,t-1}, \Delta p_{e,t}, \Delta p_{e,t-1}, \Delta U_{r,t-1}, \Delta w_{t-1}, \Delta c_{t-1}, \Delta m_{t-1}, \Delta n_{t-1}, \Delta R_{s,t-1}, \Delta R_{l,t-1}, \Delta p_{o,t-1}, I_{d,t}, \pi^*_{t-1}; \varepsilon_t),$$

where
$\pi^*_t = 0.25 e_{r,t} - 0.675(c-p)^*_t - 0.075(p_o - p)_t + 0.11 I_{2,t} + 0.25$, $(c-p)^*_t = c_t - p_t + 0.006 \times (trend - 69.5) + 2.37$, and I_d is a combination of year indicator dummies.

Model space: $2^{20} = 1,048,576$ linear combinations.

We decided to replicate it with our **BACE** packages[14].

[14] Błażejowski, M., Kufel, P., & Kwiatkowski, J. (2020). Model simplification and variable selection: A replication of the UK inflation model by Hendry (2001). *Journal of Applied Econometrics*, *35*(5), 645–652.

Calling analysis via script

```
# BMA
BMA_res = BMA_ADL_GUI(d_p, y_lags, GUMBMA, with_const, model_prior, \
prior_avg_model_size, alpha, top_ranked_model, g_prior, check_statio, \
joint, forecasts, Nrep, PercentOfburnin, MC_from_post, MPI_threads, \
verbosity, progress)
# BACE
BACE_res = BACE_GUI(d_p, y_lags, GUMBACE, with_const, model_prior, \
prior_avg_model_size, alpha, top_ranked_model, joint, forecasts, Nrep, \
PercentOfburnin, MPI_threads, verbosity, progress, installed_RAM)
```

	Variable	BACE PIP	BACE Avg. Mean	BACE Avg. SD	BMA_ADL[15] PIP	BMA_ADL[15] Avg. Mean	BMA_ADL[15] Avg. SD
Hendry's model	$I_{d.t}$	1.00	0.0380	0.0015	1.00	0.0379	0.0014
Hendry's model	$\Delta p_{e.t}$	1.00	0.2612	0.0248	1.00	0.2617	0.0236
Hendry's model	S_{t-1}	1.00	-0.9786	0.1060	1.00	-0.9696	0.1024
Hendry's model	y^d_{t-1}	1.00	0.1898	0.0381	1.00	0.1875	0.0352
Hendry's model	Δp_{t-1}	1.00	0.2818	0.0353	1.00	0.2800	0.0322
Hendry's model	π^*_{t-1}	0.99	-0.1674	0.0295	1.00	-0.1684	0.0281
Hendry's model	$\Delta R_{s.t-1}$	0.99	0.6896	0.1273	0.99	0.6903	0.1199
Hendry's model	$\Delta p_{o.t-1}$	0.99	0.0492	0.0111	0.99	0.0489	0.0106
Hendry's model	Δm_{t-1}	0.99	0.1531	0.0325	0.99	0.1575	0.0309
	U^d_{t-1}	0.71	-0.0548	0.0443	0.60	-0.0472	0.0449
	n^d_{t-1}	0.20	0.0006	0.0016	0.12	0.0004	0.0013
	$R_{l.t-1}$	0.15	0.0060	0.0217	0.09	0.0038	0.0170
	$\Delta p_{e.t-1}$	0.12	0.0030	0.0134	0.07	0.0017	0.0099
	Δn_{t-1}	0.12	0.0014	0.0063	0.06	0.0007	0.0044
	$const$	0.11	0.0001	0.0007	0.07	0.0001	0.0005
	Δw_{t-1}	0.10	-0.0001	0.0132	0.05	0.0001	0.0083
	$\Delta U_{r.t-1}$	0.10	-0.0009	0.0230	0.05	-0.0001	0.0164
	m^d_{t-1}	0.10	-0.0001	0.0043	0.05	-0.0001	0.0028
	Δc_{t-1}	0.09	0.0003	0.0104	0.05	0.0001	0.0066
	$\Delta R_{l.t-1}$	0.09	0.0012	0.0839	0.05	0.0017	0.0591

[15] Błażejowski, M., Kwiatkowski, J., & Kufel, P. (2020). BACE and BMA Variable Selection and Forecasting for UK Money Demand and Inflation with Gretl. *Econometrics*, *8*(2), 21.

Run times of gretl's BMA related packages

CPUs	**UK Inflation**[16] **without forecasts** Nrep	Run time	**with forecasts** Nrep	Run time
BACE				
1	$5 \cdot 10^5$	112	$5 \cdot 10^5$	128
4	$5 \cdot 10^5$	49	$5 \cdot 10^5$	54
20	$5 \cdot 10^5$	15	$5 \cdot 10^5$	17
BMA_ADL (with stationary restrictions)				
1	$1.5 \cdot 10^5$	1457	$1.5 \cdot 10^5$	35996
4	$1.5 \cdot 10^5$	380	$1.5 \cdot 10^5$	6829
20	$1.5 \cdot 10^5$	136	$1.5 \cdot 10^5$	3044
BMA_ADL (without stationary restrictions)				
1	$1.5 \cdot 10^5$	81	$1.5 \cdot 10^5$	32095
4	$1.5 \cdot 10^5$	54	$1.5 \cdot 10^5$	6556
20	$1.5 \cdot 10^5$	55	$1.5 \cdot 10^5$	1778

[16] All computations were performed on so-called `haavelmo` machine (located at Dipartimento di Scienze Economiche e Sociali (DiSES), Ancona, Italy) which consists on 20 Hyper-Threaded Intel® Xeon® CPU E5-2640 v4 @ 2.40GHz with 256 GB operational memory running under Debian GNU/Linux 64 bits.