Simulation-based forecasting procedure for threshold models – the gretl package

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Abstract

Basic purpose of the research is to work out forecasting simulation procedures based on several types of threshold models and to use these procedures for business cycles forecasting in EU countries as well as in EU as an integrated economy.

We formulated the following research hypotheses:

1. Simulation-based procedures generate much more effective forecasts of business cycles (depending on state of the phase) comparing to direct methods.
2. Simulation-based procedures based on *bootstrap* approach allow to evaluate forecasts accuracy in much more flexible way then classic extrapolation.

Extension

The most popular threshold model is TAR described by Tong (1990). TAR was used by Tiao and Tsay (1994), Potter (1995), Proiett (1998) for modeling asymmetry of real GNP in USA. Hansen (1997) used TAR model for modeling monthly unemployment. Very important feature of TAR model is that the threshold variable is observable. It can be the lagged endogenous variable as well as exogenous one. Threshold models can be also a basis for forecasting (see Brown and Mariano (1984), De Goojier and De Bruin (1999), Clements and Smith (1999, 2000)). De Grooijer and De Bruin (1998) used simulation procedures for generating forecasts based on SETAR model with Gaussian error term. Accuracy of simulation-based forecasts was promising in comparison to accuracy of forecasts obtained by exact methods.

Outline of the work plan

Work plan of the research constitutes of the following stages:

1. To estimate and to fit the best threshold model (i.e. testing for TAR or SETAR type of nonlinearity, determining the number of regimes, choosing the threshold variable and the transition function) for each economy.

2. To generate *bootstrap* forecasts based on TAR/SETAR models for the large number of replications.

3. To evaluate accuracy of the simulation-based methods of forecasting.

4. To implement a complete gretl package for *bootstrap* forecasting procedure based on different types of threshold models.

Numerical techniques used in forecasting

Forecasting one-step ahead using TAR models is not complicated, specially when direct methods are used. On the other hand, if mis-specification occurs then two-step ahead forecast can be problematic. Recursive forecasting lead to biased solutions and to major forecast errors in consequences. Nonlinear model can be described as:

$$y_t = g(x_{t-1}; \theta) + \varepsilon_t,$$

where $\varepsilon_t \sim iid(0, \sigma^2)$, x_t is vector of exogenous variables and θ is vector of parameters. Then, one-step forecast can be expressed as conditional expectancy:

$$y_{t+1|t} = E(y_{t+1}|x_t) = g(x_t; \theta).$$

Two-step forecast $y_{t+2|t}$ is more complicated due to the error term of ε_t and need to calculate multifold integral.

Ignoring the error term can simplify the forecast to:

$$y^S_{t+2|t} = g(x_{t+1}; \theta),$$

which was called by Tong (1990) as "skeleton" forecast. This method generates biased forecast. As a consequence we need to compute multifold integral if we want to get unbiased forecast. Such integration can be made by numerical techniques. The two-step ahead forecast calculated using *bootstrap* method is:

$$y^B_{t+2} = \frac{1}{N_B} \sum_{i=1}^{N} g(x_{t+1|t} + \hat{\eta}^{(i)}_{t+1}; \theta).$$

This formula can be generalized to forecasts for longer horizon. Forecast errors are then obtained from the set of estimated residuals of $x_{t+1} = \alpha x_t + \hat{\eta}_{t+1}$ with replacement $\hat{\eta}_{t+1} = x_{t+1} - \hat{\alpha} x_t$. Additional source of bias come from estimation of parameters andthis additional uncertainty in not explained.

The *bootstrap* method for long horizon forecasting require the huge number of replications. Forecasts computed for nonlinear models will be compared with forecasts based on linear models. Forecasting accuracy for competitive procedures will be verify by comparison of the *ex post* forecast errors.

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