# ARMA-GARCH models via Kalman filter in Gretl

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## Abstract

ARMA-GARCH models can be performed with Gretl in several ways. There are the GUI ways *GARCH* and *GARCH variants* (under *Model>Time series*), that perform AR-GARCH, and there is the "script way". This way consists generally in running the MLE estimation of an ARMA-GARCH model thought in the classic form. The state space form and the Kalman filter are normally not used. Probably this way is considered too complex and low convenient in Gretl, but it is not so. For example, an ARMA(1,1)-TGARCH(1,1) can be shaped in the following state space form:

$$y_t = \mu + x_t + \theta x_{t-1}$$
$$x_t = \phi x_{t-1} + \varepsilon_t$$
$$Q_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta Q_{t-1} + \gamma S_{t-1}^- \varepsilon_{t-1}^2$$

where $Q_t = Var(\varepsilon_t)$; $S_{t-1}^- = 1 \Leftarrow \varepsilon_{t-1} < 0, S_{t-1}^- = 0 \Leftarrow \varepsilon_{t-1} \geq 0$

This model can be filtered and estimated in Gretl via Kalman filter, but there are two pivotal points: (i) defining a suitable function which modifies $Q_t$; (ii) linking the model parameters to the likelihood function in the MLE block.
For the first point, we suggest the following function:

```
function void fq(matrix *Q, scalar sigma, scalar omega,
    scalar alpha, scalar beta, scalar gamma)
    if $kalman_t>1
       Q[1,1]=omega+alpha*$kalman_uhat^2+beta*Q[1,1]+
       gamma*($kalman_uhat<0)*$kalman_uhat^2
    else
       Q={sigma^2, 0; 0, 0}
    endif
end function
```

The second point is a false problem, because any relation can be used, for example: `Q[1,1]=omega+alpha+gamma+beta+sigma^2`.
Indeed the true relation is furnished by the function `fq` in the Kalman block:

```
kalman
    …
    statevar Q ; fq(&Q, sigma, omega, alpha, beta, gamma)
end kalman
```

This way was tested on a series of the FTSE-Mib index and the results was very similar to ones with Gretl's Gig package (Table 1).

| Table 1 | Gig | way | Kalman | way |
|---|---|---|---|---|
| *coeff. / index* | *value* | *sig* | *value* | *sig* |
| const | -0.0108 | | -0.0066 | |
| AR1 | -0.0030 | | -0.0080 | |
| omega | 0.0111 | ** | 0.0115 | ** |
| alpha | 0.0581 | *** | 0.0620 | *** |
| beta | 0.9238 | *** | 0.9196 | *** |
| Llik: | -727.81 | | -725.82 | |
| AIC: | 1465.62 | | 1461.65 | |
| BIC: | 1487.95 | | 1483.98 | |

The "Kalman way" ensures all the features of a Kalman approach: exact maximum likelihood estimates (if the heteroscedastic innovations are Gaussian), smoothing, a standard forecasting tool). Obviously, in case of non-Gaussian innovations, we get QMLE estimates.

This approach can be used for local level models with conditional hetero_ scedastic disturbances as well, and the ways for shaping heteroscedasticity are very interesting.

## Short References

Cottrell A., Lucchetti R.J., *Gretl User's Guide*

Harvey A.C. (1991), *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge University Press.